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Fuzzy Multi-criteria Decision Making associated with Risk and Confidence Attributes

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Abstract

The multicriteria decision problems involve uncertainty, it is important to incorporate different types of uncertainty in any proposed solution. In this paper, we presented fuzzy MCDM approach based on risk and confidence analysis that we believe is effective in tackling complex, ill-defined and human-oriented decision problems.

Keywords: *Fuzzy set, Multicriteria decision making (MCDM), Decision maker (DM), Risk attitudes and Confidence attribute*

1. Introduction

Multicriteria decision making (MCDM) refers to screening, prioritizing, ranking a set of alternatives (also referred to as “candidates” or “actions”) under usually independent, incommensurate or conflicting criteria. We will use the following example to illustrate the concepts and methods throughout.Example: We have to reach the airport from our home to catch an airplane. The MCDM problems here are to select an appropriate travel type from thee alternatives: car, Taxi and Train. Our criteria are price, journey time and comfort.

Buckley [1] introduced the multiple criteria ranking problem with fuzzy set approach. Carlsson [2] established the Tacking system for an MCDM problem with the help of some results from fuzzy set theory. Chen and Hwang [3] introduced the attribute decision making methods and applications for fuzzy system and Hwang and Yoon [5] also introduced attribute decision making methods and applications. Deng [4] discussed multi-criteria analysis with fuzzy pair wise comparison. Liu and Pang [6] developed the multiple criteria linguistic decision model (MCLDM) for human decision making and Liang [7] also deal on some concepts for multiple criteria using fuzzy system. Robert and Fuller [8] and Ribeiro [9] showed the development of Fuzzy multiple criteria decision making and attitude. Velton and Steward [10] developed the integrated approach for multiple criteria decision analysis. Wang and Poh [11] discussed confidence analysis of Fuzzy MCDM and Watthayu and Peng [12] also discussed network based framework for multi-criteria decision making. Yeh and Deng [13] developed new algorithm for fuzzy multi-criteria decision making and Yager [14,15] show the uncertainty for intelligent decision making under fuzzy modeling. Zhao, Zhang, Zhong and Wang [17] introduced the Bi-Level Multi-criteria Multiple Constraint Level Optimization MODELS and Its Application. Ahmed Hassan, El-Bakry and Abd Allah [18] design and analysis of Multi-Criteria Spatial Decision Support System (MC-SDSS) for Animal Production. Lei and Fan [19] introduced the consept of Image Fuzzy Enhancement Based on Self-Adaptive Bee Colony Algorithm.Nurmaini, Zaiton and Firnando [20] design the algorithm for Cooperative Avoidance Control-based Interval Fuzzy Kohonen Networks Algorithm in Simple Swarm Robots and discussed its application. Prayitno, Indrawati and Utomo [21] were inroduced the concepts of Trajectory Tracking of AR.DroneQuadrotor Using Fuzzy Logic Controller.

A MCDM problem is characterized by (a) ratings of each alternative with respect to each criterion and (b) the weights given to each criterion. Classical MCDM methods assume that the ratings of alternatives and the weights of criteria are crisp numbers. Increasingly, this is recognized as unrealistic. In this example, the decision maker (DM) will be unable to assign crisp number for the journey time a car since this value is influenced by many factors. Generally,

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uncertainties arise from unquantifiable information, incomplete information, unobtainable information and partial ignorance.Since classical MCDM methods cannot handle problems with such imprecise information, the representation and interpretation of "uncertainty" and human-related subjective preference is needed. The fuzzy set theory seems to have been the most commonly used methods.

2. General Fuzzy MCDM Approach

First we describe the general approach to fuzzy MCDM without considering risk attitudes and confidence.

2.1 Problem Formulation and Definition

A general multicriteria decision problem with m alternatives $A_i, (i = 1,2,....., m)$ and n criteria $C_j, (j = 1,2,3....n)$ can be expressed as:

$$D = [x_{ij}] \text{ and } W = [w_{ij}] \quad (1)$$

where $i = 1,2,....., m$ and $j = 1,2,3....n$.

Here D is referred to as the decision matrix (where the entry x_{ij} represents the rating of alternatives A_i with respect to the criterion C_j), and W as the weight vector (where w_{ij} represents the weight of criterion C_j). In general we classify criteria as either:

- Benefit criteria (where the higher the value of x_{ij} the better it is for the DM)
- Cost criteria (where the lower the value of the x_{ij} the better it is for DM).

Because we wish to consider fuzzy, as opposed to crisp, value in D and W we shall use the notation:

$$\tilde{D} = [\tilde{x}_{ij}] \text{ and } \tilde{W} = [\tilde{w}_{ij}] \quad (2)$$

Where $\tilde{x}_{ij}$ represents the fuzzy rating of alternatives A_i with respect to the criterion C_j, and $\tilde{w}_{ij}$ represents the fuzzy weight of criterion C_j. In particular, an intuitively easy and effective approach to capturing the expert's uncertainty about the value of unknown number is a triangular fuzzy number.

Definition: A triangular fuzzy number $\tilde{a}$ is defined by a triplet (a_1, a_2, a_3). The membership function is defined as:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{(x - a_1)}{(a_2 - a_1)} & a_1 \le x \le a_2 \\ \dfrac{(a_3 - x)}{(a_3 - a_2)} & a_2 \le x \le a_3 \\ 0 & otherwise \end{cases} \quad (3)$$

The triangular fuzzy number is based on a three-value judgment: the minimum possible value of a_1, the most possible value of a_2 and the maximum value a_3

Decision matrix and weight vector (Table-A)

	Price(Rs.,0.3)	Journey Time(min; 0.5)	Comfort([1,10]; 0.2)
Car	(9, 10, 12)	(70, 100, 120)	(4, 5, 6)
Taxi	(20, 24, 25)	(60, 70, 100)	(7, 8, 10)
Train	(15, 15, 15)	(70, 80, 90)	(1, 4, 7)

The above table shows the decision matrix and weight vector for the travel problem introduced. In this example the criteria price and journey time are cost criteria measured in Rupees and minutes respectively. The criterion comfort is a value criterion measured on a scale 1 to 10. The ratings in the decision matrix are expressed in triangular fuzzy number (so, for example, the car journey to the airport most typically costs Rs. 10 but it can be as low as 9 and as high as 12). For simplicity the weights are crisp numbers summing to1. (Usually the Dm is able to express the weights in this way).

2.2 Normalization

To deal with criteria on different scales, we apply a normalization process. Specifically, we normalize the fuzzy numbers in the decision matrix as the performance matrix:

$$\tilde{P} = [\tilde{P}_{ij}] \tag{4}$$

where

$$\tilde{P}_{ij} = \begin{cases} \left(\dfrac{x_{ij1}}{M}, \dfrac{x_{ij2}}{M}, \dfrac{x_{ij3}}{M}\right) & M = \max_i x_{ij3}, C_j \text{ is benefit criterion} \\ \left(\dfrac{N - x_{ij3}}{N}, \dfrac{N - x_{ij2}}{N}, \dfrac{N - x_{ij1}}{N}\right) & N = \max_i x_{ij3}, C_j \text{ is cost criterion} \end{cases}$$

This method preserves the ranges of normalized triangular fuzzy numbers to [0, 1].

Example: The performance matrix for the decision matrix of table-A calculated by eqn (4) is given by:

Performance matrix (Table-B)

	Price(Rs.,0.3)	Journey Time(min; 0.5)	Comfort([1,10]; 0.2)
Car	(0.52, 0.6, 0.64)	(0.0, 0.167, 0.417)	(0.4, 0.5, 0.6)
Taxi	(0.0, 0.04, 0.2)	(0.167, 0.417, 0.5)	(0.7, 0.8, 1.0)
Train	(0.4, 0.4, 0.4)	(0.25, 0.333, 0.417)	(0.1, 0.4, 0.7)

2.3 Weighting the Criteria

We construct the weighted performance matrix by multiplying the weight vector by the decision matrix as:

$$\tilde{P}^w = [\tilde{P}_{ij}^w] \tag{5}$$

where $P_{ij1}^w = w_{j1}, P_{ij2}^w = w_{j2}, P_{ij3}^w = w_{j3}, \ldots\ldots\ldots$ for $i = 1,2,\ldots\ldots, m$ and $j = 1,2,3\ldots.n$.

Example: Weighted Performance matrix (Table-C)

	Price(Rs.,0.3)	Journey Time(min; 0.5)	Comfort([1,10]; 0.2)
Car	(0.156, 0.18, 0.192)	(0.0, 0.0835, 0.2084)	(0.08, 0.1, 0.12)
Taxi	(0.0, 0.012, 0.06)	(0.0835, 0.2084, 0.25)	(0.14, 0.16, 0.2)
Train	(0.12, 0.12, 0.12)	(0.125, 0.1665, 0.2084)	(0.02, 0.08, 0.14)

2.4 Performance Alternatives

The most preferred alternatives should have the shortest distance form the positive ideal solution and the longest distance from the negative ideal solution.

Definition: Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two positive triangular fuzzy numbers, then the vertex method defines the distance between them as:

$$d(\tilde{a}, \tilde{b}) = \left\{ \frac{\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 \right]}{3} \right\}^{\frac{1}{2}} \tag{6}$$

For the normalized fuzzy performance matrix, we define the positive ideal solution $\tilde{P}_j^+ = (1,1,1)$ and the negative ideal solution $\tilde{P}_j^- = (0,0,0)$ under criteria. By the vertex method, the distance between each alternative and the positive ideal solution and negative ideal solution is calculated as:

$$d_i^+ = \sum_{j=1}^{n} d(\tilde{P}_{ij}^w, \tilde{P}_j^+) \tag{7}$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{P}_{ij}^w, \tilde{P}_j^-) \tag{8}$$

where $i = 1,2,....., m$ and $j = 1,2,3....n$

We calculate performance index for each alternatives as:

$$p_i = d_i^- + n - d_i^+ / 2n \tag{9}$$

where $i = 1,2,....., m$ and n is the number of criteria. The nearer p_i gets to 1, the better, the alternative's performance.

Example: By using eqn (7) and (8) we can calculate the alternative distance to the positive ideal and negative ideal solution and by using eqn (9) we can calculate the alternatives' performance index together with ranking orders

Performance Index (Table-D)

Car		Taxi		Train	
P	Order	P	Order	P	Order
0.1294	1	0.1275	2	0.1248	3

3. Fuzzy MCDM by Incorporating Risk Attitudes

The general approach can provide a basic ranking of the alternatives, but it cannot deal with DM's attitudes towards risk and uncertainty. Here we explain how to incorporate the DM's risk attitude into the general fuzzy MCDM approach. The linguistic approach to modeling risk attitudes in fuzzy MCDM uses the notation of "optimism" and "pessimism". The key issue for us is to be able to use natural language to describe an appropriate range of attitude between the extremes of “optimism” and “pessimism”. The number of terms needs to be small enough so as not to impose pointless precision, yet rich enough to allow proper discrimination of the assessments. Based on the Miller’s theory of cognitive retention we use nine as the maximum number of terms for the DM’s assessment.

3.1 Modeling Risk Attitude

For benefit criteria, the DM expects a maximum value as the best value. For cost criteria, the DM expects a minimum value as the best value. To incorporate the DM’s risk attitude, to the triangular fuzzy number (a_1, a_2, a_3), as the neutral attitudes (a_1, a_3, a_3) and (a_1, a_1, a_3) as absolutely optimistic (AO) (absolutely pessimistic) (AP)) and absolutely pessimistic (AP) (absolutely optimistic (AO)) for benefit (cost) criteria. In general we use an ordered structure to incorporate other risk attitudes in (a_1, a_2, a_3) according to benefit (cost) criteria as shown in the table-E. The first column shows the set of linguistic terms. In case of benefit criteria in the second column and cost criteria in the third column shows the associated triangular fuzzy numbers derived from the triangular fuzzy number (a_1, a_2, a_3). The approach described here is easily generalized for the case where there are n as opposed to 9 linguistic terms.

Linguistic Terms of Risk Attitude (Table-E)

Linguistic Term	Triangular Fuzzy Number Derived from (a_1, a_2, a_3) for benefit criteria	Triangular Fuzzy Number Derived from (a_1, a_2, a_3) for cost criteria
Absolutely Optimistic(AO)	(a_1, a_3, a_3)	(a_1, a_1, a_3)
Very Optimistic(VO)	$(a_1, (a_2 + 3a_3)/4, a_3)$	$(a_1, (a_2 + 3a_1)/4, a_3)$
Optimistic (O)	$(a_1, (a_2 + a_3)/2, a_3)$	$(a_1, (a_2 + a_1)/2, a_3)$
Fairly Optimistic(FO)	$(a_1, (3a_2 + a_3)/4, a_3)$	$(a_1, (3a_2 + a_1)/4, a_3)$
Neutral (N)	(a_1, a_2, a_3)	(a_1, a_2, a_3)
Fairly Pessimistic(FP)	$(a_1, (3a_2 + a_1)/4, a_3)$	$(a_1, (3a_2 + a_3)/4, a_3)$
Pessimistic(P)	$(a_1, (a_2 + a_1)/2, a_3)$	$(a_1, (a_2 + a_3)/2, a_3)$
Very Pessimistic(VP)	$(a_1, (a_2 + 3a_1)/4, a_3)$	$(a_1, (a_2 + 3a_3)/4, a_3)$
Absolutely Pessimistic(AP)	(a_1, a_1, a_3)	(a_1, a_3, a_3)

3.2 Performance of Alternative on Risk Attitudes

Now that we have triangular fuzzy numbers that capture the DM’s risk attitude we incorporate these into the decision matrix as:

$$\tilde{D}' = [\tilde{x}'_{ij}] \quad (10)$$

where $\tilde{x}'_{ij}$ is the triangular fuzzy number derived from $\tilde{x}_{ij}$ under the specific risk attitude from table-E. After normalization and weighting of criteria, we obtain the performance index with respect to the risk attitude.

Performance index with respect to risk Attitudes (Table-F)

Term	Car		Taxi		Train	
	P	Order	P	Order	P	Order
AO	0.1465	1	0.1418	2	0.1363	3
VO	0.1419	1	0.1380	2	0.1333	3
O	0.1375	1	0.1343	2	0.1303	3
FO	0.1333	1	0.1308	2	0.1275	3
N	0.1294	1	0.1275	2	0.1248	3
FP	0.1263	1	0.1233	2	0.1221	3
P	0.1235	1	0.1193	3	0.1196	2
VP	0.1208	1	0.1154	3	0.1173	2
Ap	0.1183	1	0.1119	3	0.1151	2

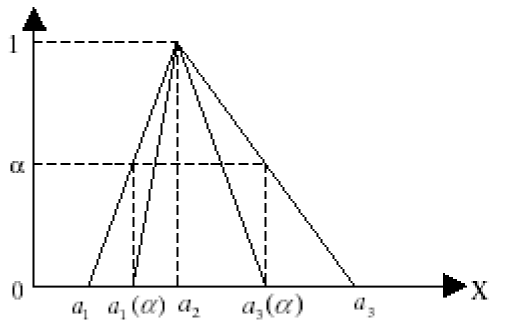
4. Fuzzy MCDM by Incorporating Confidence Attitudes

From table-F it suggests that any Dm ranging from an extreme optimistic to extreme pessimistic will always choose the car as the preferred alternatives. However, this result does not take account of the DM's confidence/uncertainty about the value of rating. Fro example, the fuzzy value of journey time for car is (70, 100, 120) compared with (70, 80, 90) for train. Somebody who was extremely confident about the values would tend to believe that the most likely value was the true value in each case, i.e. 100 and 80 respectively. Thus a pessimistic who was nevertheless extremely confident about the value would be more likely to favor the train than the car. Here we formalize these notations so that we are able to complete our MCDM process by incorporating the DM's confidence on top of their risk attitudes.

4.1 Incorporating Confidence Level

To assess confidence and uncertainty about a triangular fuzzy number we use $\alpha - cuts$ concept. The idea is that $\alpha \in [0,1]$ is a basic measure of our confidence about the fuzzy number. We use it to compute a refined fuzzy number that is 'closer' to the value with highest possibilities as α tends to 1. Formally, assuming that the confidence in the triangular fuzzy number $\tilde{a} = (a_1, a_3, a_3)$ is at level α, the refined fuzzy number is defined as:

$$\tilde{a}^{\alpha} = (a_1(\alpha), a_2, a_3(\alpha)) = (a_1 + \alpha(a_2 - a_1), a_3, a_3 - \alpha(a_3 - a_3)) \quad (11)$$



A triangular fuzzy number Ā and its α-cut triangular fuzzy number.

Having already incorporated the risk attitude in the decision matrix, we can now construct the decision matrix with risk attitude given confidence level α as:

$$\tilde{D}^{\alpha} = [\tilde{x}_{ij}^{\alpha}] \quad (12)$$

where $\tilde{x}_{ij}^{\alpha}$ is the triangular fuzzy number derived from $\tilde{x}'_{ij}$ under the specific confidence level by eqn (11). Suppose that there are l confidence levels. After normalization and weighting of criteria, we obtain the performance index vector given confidence level as:

$$P_i = P_{ik}^{\alpha}\text{, where } \alpha = k-1/l-1, (l \geq 2) \tag{13}$$

where $i = 1,2,3,\ldots\ldots, m$ and $k = 1,2,3,\ldots\ldots, l$
By applying this equation we get the performance index under neutral risk attitude with 11 confidence level.

Performance index under neutral risk attitude with 11 confidence level (Table-G)

	Confidence Level										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Car	0.1294	0.1269	0.1243	0.1217	0.1189	0.1160	0.1131	0.1100	0.1068	0.1035	0.1000
Taxi	0.1275	0.1264	0.1252	0.1240	0.1229	0.1217	0.1206	0.1196	0.1185	0.1176	0.1167
Train	0.1248	0.1228	0.1208	0.1188	0.1167	0.1146	0.1126	0.1105	0.1084	0.1063	0.1042

4.2 Modeling of Confidence Attitudes

Instead of providing a direct value α to construct a confidence level, we next use linguistic variables to represent the DM's qualitative assessment of confidence. As before, we use nine point linguistic scales as shown in the table-H. Intuitively the membership value of the confidence increases linearly as α increases from 0 to 1.

Linguistic Terms of Confidence Attitudes (Table-H)

Linguistic Term	Membership Function
Absolutely Confident(AC)	$\mu_{AC}(\alpha) = \begin{cases} 1 & \alpha = 1 \\ 0 & otherwise \end{cases}, \alpha \in [0,1]$
Very Confident(VC)	$\mu_{VC}(\alpha) = (\mu_C(\alpha))^2 = (\alpha)^2, \alpha \in [0,1]$
Confident (C)	$\mu_C(\alpha) = \alpha, \alpha \in [0,1]$
Fairly Confident(FC)	$\mu_{VC}(\alpha) = (\mu_C(\alpha))^{0.5} = \sqrt{\alpha}, \alpha \in [0,1]$
Neutral (N)	$\mu_N(\alpha) = 1, \alpha \in [0,1]$
Fairly Non- Confident (FNC)	$\mu_{FNC}(\alpha) = (1-\mu_C(\alpha))^{0.5} = \sqrt{1-\alpha}, \alpha \in [0,1]$
Non- Confident (NC)	$\mu_{NC}(\alpha) = 1-\mu_C(\alpha) = 1-\alpha, \alpha \in [0,1]$
Very Non- Confident (VNC)	$\mu_{VNC}(\alpha) = (1-\mu_C(\alpha))^2 = (1-\alpha)^2, \alpha \in [0,1]$
Absolutely Non- Confident (ANC)	$\mu_{ANC}(\alpha) = \begin{cases} 1 & \alpha = 0 \\ 0 & otherwise \end{cases}, \alpha \in [0,1]$

5. Performance of Alternatives on confidence Attitudes

In general, assuming a total of $l(l \geq 2)$ confidence levels, we define the normalized confidence membership vector as:

$$C_{LT} = \left(\frac{c_k}{\sum_{k=1}^{l} c_k} \right) \tag{14}$$

where $c_k = \mu_{LT}(\alpha), \alpha = k-1/l-1, k = 1,2,3,....., l$ and LT represents linguistic terms AC, VC, C, FC, N, FNC, NC, VNC, AND ANC, respectively. Based on the confidence membership vectors, the performance of ith alternative under confidence attitude is:

$$P_i^{LT} = P_i(C_{LT})^T = \frac{\sum_{k=1}^{l} p_{ik} c_k}{\sum_{k=1}^{l} c_k} \tag{15}$$

The DM can rank, prioritize and select alternatives under different attitudes and confidence attitudes according to the performance index.

Performance Index under Neutral Risk Attitude With Respect to Confidence Attitude (Table-I)

	P	Order	P	Order	P	Order
AC	0.1000	3	0.1167	1	0.1042	2
VC	0.1071	3	0.1189	1	0.1088	2
C	0.1085	3	0.1185	1	0.1093	2
FC	0.1110	3	0.1197	1	0.1112	2
N	0.1143	2	0.1207	1	0.1134	3
FNC	0.1189	2	0.1227	1	0.1167	3
NC	0.1202	2	0.1228	1	0.1175	3
VNC	0.1239	2	0.1252	1	0.1206	3
ANC	0.1294	1	0.1275	2	0.1248	3

For clear evaluation and analysis, we calculate and show the alternatives performance index under risk and confidence attitude simultaneously. Performance index and ranking order of car, taxi and train under different risk and confidence attitudes are shown in the following table.

Performance Index of Car under risk and confidence Attitudes (Table-J)

Car	AO	VO	O	FO	N	FP	P	VP	AP
AC	0.0950	0.0963	0.0976	0.0988	0.1000	0.0976	0.0951	0.0926	0.0901
VC	0.1101	0.1091	0.1083	0.1076	0.1071	0.1042	0.1014	0.0987	0.0960
C	0.1138	0.1122	0.1108	0.1095	0.1085	0.1056	0.1028	0.1000	0.0973
FC	0.1179	0.1158	0.1140	0.1124	0.1110	0.1080	0.1051	0.1023	0.0995
N	0.1238	0.1211	0.1186	0.1163	0.1143	0.1113	0.1084	0.1056	0.1029
FNC	0.1314	0.1279	0.1246	0.1216	0.1189	0.1157	0.1127	0.1098	0.1027
NC	0.1338	0.1300	0.1264	0.1231	0.1202	0.1170	0.1140	0.1111	0.1084
VNC	0.1389	0.1348	0.1309	0.1272	0.1239	0.1207	0.1177	0.1148	0.1121
ANC	0.1465	0.1419	0.1375	0.1333	0.1294	0.1263	0.1235	0.1208	0.1183

Ranking Order of Car under Risk and Confidence Attitudes (Table-K)

Car	AO	VO	O	FO	N	FP	P	VP	AP
AC	1	2	2	3	3	3	3	3	3
VC	1	2	2	2	3	3	3	3	3
C	1	1	2	2	3	3	3	3	3
FC	1	1	2	2	3	3	3	3	2
N	1	1	2	2	2	2	2	1	1
FNC	1	1	1	2	2	2	2	1	1
NC	1	1	1	2	2	2	1	1	1
VNC	1	1	1	1	2	1	1	1	1
ANC	1	1	1	1	1	1	1	1	1

Performance Index of Taxi under risk and confidence Attitudes (Table-L)

Taxi	AO	VO	O	FO	N	FP	P	VP	AP
AC	0.0905	0.0989	0.1059	0.1117	0.1167	0.1103	0.1045	0.0993	0.0944
VC	0.1050	0.1094	0.1131	0.1162	0.1189	0.1129	0.1073	0.1022	0.0973
C	0.1087	0.1119	0.1145	0.1166	0.1185	0.1127	0.1073	0.1023	0.0976
FC	0.1127	0.1150	0.1169	0.1185	0.1197	0.1141	0.1088	0.1038	0.0992
N	0.1187	0.1195	0.1201	0.1204	0.1207	0.1153	0.1103	0.1055	0.1011
FNC	0.1263	0.1255	0.1245	0.1236	0.1227	0.1176	0.1128	0.1082	0.1039
NC	0.1287	0.1272	0.1257	0.1242	0.1228	0.1179	0.1132	0.1088	0.1046
VNC	0.1339	0.1316	0.1293	0.1272	0.1252	0.1204	0.1158	0.1115	0.1074
ANC	0.1418	0.1380	0.1343	0.1308	0.1275	0.1233	0.1193	0.1154	0.1119

Ranking Order of Taxi under Risk and Confidence Attitudes (Table-M)

Car	AO	VO	O	FO	N	FP	P	VP	AP
AC	2	1	1	1	1	1	1	1	1
VC	2	1	1	1	1	1	1	1	1
C	2	2	1	1	1	1	1	1	2
FC	2	2	1	1	1	1	1	1	3
N	2	2	1	1	1	1	1	2	3
FNC	2	2	2	1	1	1	1	3	3
NC	2	2	2	1	1	1	2	3	3
VNC	2	2	2	2	1	2	2	3	3
ANC	2	2	2	2	2	2	3	3	3

Performance Index of Train under risk and confidence Attitudes (Table-N)

Taxi	AO	VO	O	FO	N	FP	P	VP	AP
AC	0.0717	0.0832	0.0922	0.0991	0.1042	0.1018	0.0989	0.0953	0.0912
VC	0.0903	0.0968	0.1019	0.1059	0.1088	0.1062	0.1034	0.1002	0.0967
C	0.0954	0.1004	0.1043	0.1072	0.1093	0.1068	0.1040	0.1010	0.0977
FC	0.1002	0.1042	0.1073	0.1096	0.1112	0.1086	0.1059	0.1029	0.0988
N	0.1079	0.1102	0.1118	0.1129	0.1134	0.1108	0.1081	0.1053	0.1025
FNC	0.1172	0.1175	0.1175	0.1172	0.1167	0.1141	0.1115	0.1088	0.1062
NC	0.1204	0.1199	0.1193	0.1185	0.1175	0.1149	0.1123	0.1097	0.1072
VNC	0.1264	0.1251	0.1236	0.1221	0.1206	0.1179	0.1153	0.1128	0.1104
ANC	0.1363	0.1333	0.1303	0.1275	0.1248	0.1221	0.1196	0.1173	0.1151

Ranking Order of Train under Risk and Confidence Attitudes (Table-O)

Car	AO	VO	O	FO	N	FP	P	VP	AP
AC	3	3	3	2	2	2	2	2	2
VC	3	3	3	3	2	2	2	2	2
C	3	3	3	3	2	2	2	2	1
FC	3	3	3	3	2	2	2	2	1
N	3	3	3	3	3	3	3	3	2
FNC	3	3	3	3	3	3	3	2	2
NC	3	3	3	3	3	3	3	2	2
VNC	3	3	3	3	3	3	3	2	2
ANC	3	3	3	3	3	3	2	2	2

Thus Dm can choose the best alternative under different risk and confidence attitudes accordingly. For example, a Dm who is pessimistic (with respect to risk attitude) but very confident will rank car as the last alternative and taxi is the first, whereas the Dm who is absolutely pessimistic and fairly confident will rank train as first.

6. Conclusion

In this paper our approach consists of the following: formulate the problem in terms of fuzzy decision matrix and the weight vector, normalize the decision matrix as the performance matrix., construct the weighed performance matrix., with reference to ideal solutions, calculate alternatives performance index, according to DM's risk attitudes (which can be characterized linguistically), construct the performance matrix with risk attitudes. Calculate alternatives performance index under risk attitudes, construct the performance matrix with risk attitudes on confidence levels and calculate performances index vector with respect to confidence levels, according to DM's confidence attitudes (which can again be characterized linguistically), determine the confidence membership vectors and calculate alternatives performance index under confidence attitudes.

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