Synthesis of sliding mode control for flexible-joint manipulators based on serial invariant manifolds

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ABSTRACT

This paper focuses on synthesizing sliding mode control (SMC) for flexible-joint manipulators (FJM) based on serial invariant manifolds in order to increase the control quality for the system. SMC based on the serial invariant manifolds is proposed. The control law is found based on synergetic control theory (SCT) and analytical design of aggregated regulators (ADAR) method. In order to improve the control quality due to the effect of the stiffness value between two links in the system, a mechanism for constructing manifolds is built. The time response of the outer loop manifolds close to the actuator will be larger in the next round. The control quality of the system can be pre-evaluated through the parameters of the designed manifolds. Global stability is demonstrated by using the Lyapunov function in the design process. Finally, the effectiveness of the proposed controller based on SCT is demonstrated by numerical simulation results and compared with the traditional SMC.

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1. INTRODUCTION

The development and application of technical manipulators led to many studies of manipulator control problems. Flexible-joint manipulators (FJM) offer several advantages over rigid-joint manipulators, such as light weight, lower cost, smaller actuators, larger workloads, mechanical capabilities, better mobility and transport, higher operating speeds, energy efficiency, and a larger number of applications. Therefore, they are often operated at high speeds to provide high productivity. However, when the control in the joint space operates using only the motor drive feedback, which is the most common case in robotics, the relative joint torsion is uncompensated lead to position errors under heavy load and large joint torque [1], [2]. In some cases, joint flexibility can lead to instability when neglected in control design, as explained in [3]. Non-linearity, model uncertainty, friction, perturbation, and noise effects further complicate model-based controller design in controllers of FJM. Therefore, the conflicting requirements between high speed and high precision make for a challenging control problem.

Research on dynamic modeling and control of FJM have received increasing attention in the last decades [1]–[22]. Many manipulators incorporate harmonic actuators to reduce speed, and it is known that such actuators exert elastic force into the joints. Industrial manipulators generally have elastic elements in the drive system, which can lead to the appearance of strong oscillations. For multi-degrees of freedom manipulators, joint elasticity can arise from a number of sources, such as elasticity in gears, belts, tendons, bearings, hydraulic lines, and can limit the speed and be obtained dynamic accuracy by designed control algorithms and assuming perfect stiffness at the joints. Along with some of the benefits and applications of a flexible-joint mechanism, joint elasticity presents a number of control challenges. Nonlinearities, model uncertainties, friction, perturbations, and noise effects further complicate model-based controller design in flexible-joint controllers [2]. Attempts to address the dynamic challenges of general flexibility have resulted in several control strategies being adopted to control these systems.

Although considerable research has been performed on the dynamics and control of FJM, most of the currently available flexible-joint adaptive control strategies reported in the literature are techniques based on the model. The control problem of FJM is to design the controller so that the linkage of the manipulator can reach the desired position or follow precisely the specified trajectory with minimal vibration to the link. To achieve these goals, various methods using different techniques have been proposed as follows: the flexible-joint controller model was first proposed in the study [9]. Since then, many control methods have been applied to improve the tracking performance of the FJM, such as proportional integral derivative (PID) control [6], [7], linear quadratic regulator (LQR). [17], fuzzy logic control [13], [14], sliding mode control (SMC) [4], [5], backstepping control [15], robust control [11] and neural network control [5], [6]. For example, in [6], the authors designed a PID controller for FJM when considering it similar to rigid-joint manipulators and proved its efficiency through simulation results. Agee *et al*. [7], presented iPI controller to improve control quality when the object model is incomplete and has external disturbances, the results are proven through simulation and experiment results. But the essence of the PID controller and its variants is that the control signal is based on error information. Therefore, regardless of the physical nature of the object, it may not give the best control quality to the system. Doina [17], presented the use of LQR controller for this system. The results show that the effectiveness of the method is the applicability on embedded systems of the LQR controller. But the design of the LQR controller must conduct linearization of the control object, leading to the controller not having a good response when the system is far from the working point, and determining the matrices in the objective function is also difficult.

SMC is one of the most powerful and well-regarded control techniques for nonlinear systems [3]–[6]. It can be applied to FJM and provides robust control to against disturbances and uncertainty of model [3]. The SMC provides the high accuracy and rapid system dynamic response in feedback loops, monitoring or tuning modes, as well as robustness to parameter variations and external disturbances [5]. The principle of traditional SMC is to limit the system trajectory to a sliding surface and then the system moves on sliding surface to the desired point. The selection of the appropriate sliding function represents an important part of the SMC design to stabilize the system trajectory.

In this study, SMC law is designed based on serial invariant manifolds combined with synergetic control theory (SCT) [23]–[32]. The first invariant manifold is designed from a state under direct action of the actuator, and once the system has fallen into that manifold, the system becomes internally stable. The process of determining the internal control signal continues based on variable manifolds, which are intended to bring the system to the desired point or trajectory. In SCT, these points and trajectories are called technical invariants or control targets. SMC law and the internal control signal are found based on the analytical design of aggregated regulators (ADAR) method. The use of manifolds and the ADAR method allows us to tune the control quality through the physical nature of the object. In addition, the control law is found to ensure that the system is globally asymptotically stable at the first step. The quality of the system response of this method is shown by simulation results on MATLAB software and the effectiveness of the method is shown by comparison with the traditional SMC.

2. MATHEMATICAL MODEL OF FJM

The FJM considered in this paper is shown in Figure 1, where q_1 is the rotation angle of the link of the flexible joint and q_2 is the position of the motor shaft rotation angle. The purpose of the controller is to generate moment on the shaft. This moment through the flexible joint will act on the link to stabilize or to track a given trajectory. The difference of the flexible joint response is determined by the spring's elasticity as well as its intrinsic physical properties [4], [8]. The elasticity of the joint is described by the stiffness K of a linear torsion spring. Parameters I and J are the link inertia and motor inertia, respectively, and l is the height of the center of the link block. The equation of motion for this system is obtained using the Euler-Lagrange equation where L is the sum of kinetic energy K_{tot} and potential energy P_{tot}, which are defined as follows:

$$K_{tot} = \frac{1}{2} I \dot{q}_1^2 + \frac{1}{2} J q_2^2 \tag{1}$$

$$P_{tot} = \frac{1}{2} k (q_1 - q_2)^2 + mgl\, cos(q_1) \tag{2}$$

$$L = K_{tot} + P_{tot} \tag{3}$$

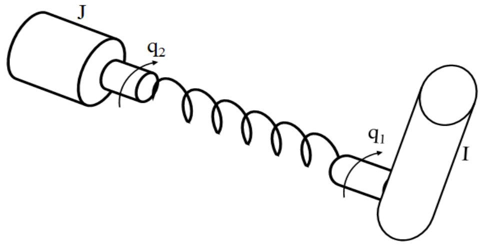

Figure 1. Flexible joint manipulator

Using the Euler-Lagrange equation of motion (4) for the variables q_1 and q_2, we get the dynamic equations of the system as (4):

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \frac{\partial L}{\partial q_1} = 0 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_2}\right) - \frac{\partial L}{\partial q_2} = \tau \end{cases} \tag{4}$$

in (4), τ represents the torque or control force generated by the actuator.

$$\begin{cases} I\ddot{q}_1 + mgl\,sin(q_1) + k(q_1 - q_2) = 0 \\ J\ddot{q}_2 - k(q_1 - q_2) = \tau \end{cases} \tag{5}$$

Set the state variables as follows: $[q_1\ \dot{q}_1\ q_2\ \dot{q}_2]^T = [x_1\ x_2\ x_3\ x_4]^T$, the equation of the FJM (5) can be written as a state space model as (6):

$$\begin{cases} \dot{x}_1 = x_2; \qquad \dot{x}_2 = -\frac{mgl}{I} sin\ (x_1) - \frac{k}{I}(x_1 - x_3); \\ \dot{x}_3 = x_4; \qquad \dot{x}_4 = \frac{k}{J}(x_1 - x_3) + \frac{\tau}{J} \end{cases} \tag{6}$$

The control objective is firstly to ensure that the state of the system changes stably to a desired operating point and that the static error approaches zero as time approaches infinity. To make it easier to write mathematical models in future calculations, we add the following functions:

$$\begin{vmatrix} f_1(x_1, x_2, x_3) = -\frac{mgl}{I} sin(x_1) - \frac{k}{I}(x_1 - x_3) \\ f_2(x_1, x_2, x_3) = \frac{k}{J}(x_3 - x_1) \end{vmatrix} \tag{7}$$

according to (8) system (7) can be written as:

$$\begin{cases} \dot{x}_1 = \dot{x}_2; \qquad \dot{x}_2 = f_1(x_1, x_2, x_3); \\ \dot{x}_3 = x_4; \qquad \dot{x}_4 = f_2(x_1, x_2, x_3) + \frac{1}{J}\tau \end{cases} \tag{8}$$

here the parameters of the model used when simulating the control law are given in Table 1.

Table 1. The parameter of the FJM

Symbol	Discription	Value	Unit
m	Mass of link	1.0	*kg*
k	Stiffness	50	*Nm/rad*
J	Inertia of motor actuator	1	*kg m2*
I	Inertia of flexible link	1	*kg m2*
g	Gravity	9.81	*m/s2*
l	Length of flexible link	1	*m*
m	Mass of link	1.0	*kg*

3. CONTROLLER DESIGN

3.1. Sliding control synthesis method based on invariant manifold sequential set

When it is not possible to immediately specify a sliding surface of a manifold for the synthesis of nonlinear SMCs in SCT, a series or parallel set of invariant manifolds in phase space can be used based on the developed scalar and vector synthesis [23], [24]. A descriptive review of the synthesis method of SMC based on serial invariant manifolds has been presented in the study [3]. Assume that the initial differential equation of the control object has the form (9):

$$\begin{cases}\dot{x}_i(t) = f_i(x_1, \dots, x_n) + a_{i+1}x_{i+1}, \; i = \overline{1, n-1} \\ \dot{x}_n(t) = f_n(x_1, \dots, x_n) + u\end{cases} \tag{9}$$

where $\boldsymbol{x}=[x_1, \dots, x_n]^T$ is a vector of state variables, dim(x) = n×1; $u=u(\boldsymbol{x})$ - a scalar control signal, and $f_i(x_1, \dots, x_n)$; $i = \overline{1, n-1}$ is a continuously differentiable function.

For system (9), the problem of SMC synthesis is posed: it is necessary to define a control law $u(x)$-a function of the state variables of the object (1), ensuring the object transition from an arbitrary initial state (in some acceptable area) to a certain state determined by the desired invariant manifolds-the control target.

In the first step of the synthesis a manifold will be considered.

$$\psi_1 = \sum_{k=1}^{n-1} \beta_k |x_k| + |s_1| = 0 \tag{10}$$

With $s_1 = x_n - \varphi_1(x_1, \dots, x_{n-1}) = 0$, where $\varphi_1(x_1, \dots, x_{n-1})$ is an unknown continuous function at this step, as an internal control for the decomposed system of the next stage:

$$\begin{cases}\dot{x}_i(t) = f_i(x_1, \dots, x_n) + a_{i+1}x_{i+1}, \quad i = \overline{1, n-2} \\ \dot{x}_{n-1}(t) = f_{n-1}(x_1, \dots, x_{n-1}) + a_n\varphi_1(x_1, \dots, x_{n-1})\end{cases} \tag{11}$$

based on the objective function of SCT and ADAR method.

$$T_1\dot{\psi}_1 + \psi_1 = 0 \tag{12}$$

From the original equations of the object (9), the manifold (10) and (12) find the desired control law.

$$u = -\left(\sum_{k=1}^{n-1} \beta_k (f_k(x_1, \dots, x_n) + a_{k+1}x_{k+1}) sign(x_k) + \frac{1}{T_1}\psi_1\right) sign(s_1) - \\ - \sum_{i=1}^{n-1} \frac{\partial \varphi_1}{\partial x_i}(f_i(x_1, \dots, x_n) + a_{i+1}x_{i+1}) - f_n(x_1, \dots, x_n). \tag{13}$$

This control signal moves the system from an arbitrary initial state to a manifold $\psi_1=0$. Since the root $\psi_1=0$ of (12) is asymptotically stable when $T_1>0$, this means that the system state falls on the submanifold s_1, that is, on the sliding surface. The steady motion along $s_1=0$ can be organized using submanifolds $s_2, \dots, s_m$:

$$\begin{cases} s_2 = x_{n-1} - \varphi_2(x_1, \dots, x_{n-2}) = 0; \\ \dots \\ s_{n-1} = x_2 - \varphi_{n-1}(x_1) = 0; \end{cases} \tag{14}$$

and synthesize intermediate control laws $\varphi_2, \dots, \varphi_{n-1}$ on the basis of functional equations of the form:

$$T_i\dot{s}_i + s_i = 0, \quad i = \overline{2, n-1}; \; T_i > 0 \tag{15}$$

and the decomposed system has the form (11).

3.2. Synthesis method of SMC based on serial invariant manifold for FJM

The purpose of the flexible-joint control is to ensure that the link q_1 moves in the desired trajectory x_d by changing the voltage supplied to the motor to create a torque u acting on the motor shaft. From the point of view of SCT, it is necessary to synthesize the control signal $\tau(x_1, x_2, x_3, x_4)$ - a function that depends on the phase coordinates. The control signal will move flexible-links from the initial position following a given signal or stabilize at the desired position to ensure the required quality of the system when there are effects of disturbances.

From the purpose of the control problem, FJM is stable at a given desired position based on SCT, the authors propose the first invariant manifold corresponding to the control target.

$$x_1 = x_d \tag{16}$$

In the first step, based on the practical and mathematical model of the system, when the control signal u changes, it will affect on the dynamics of links q_1 and link q_2. So that the first invariant manifold is selected as:

$$\psi_1 = |s_1| = 0 \tag{17}$$

where $s_1 = x_4 - \varphi_1(x_1, x_2, x_3)$. The function $\varphi_1(x_1, x_2, x_3)$ determines the desired characteristics of the changing velocity of q_1 at the intersection with the invariant manifold $\psi_1 = 0$. The function $\varphi_1(x_1, x_2, x_3)$ is determined in the process of synthesizing the control law, proceeding from the invariant condition (14). To ensure the manifold (15) is globally stable, according to ADAR [25]–[27], the macro variable ψ_1 must satisfy the roots of the basic (18):

$$T_1\dot{\psi}_1 + \psi_1 = 0 \tag{18}$$

where $T_1>0$ ensures the asymptotic stability of the system. Substituting (17) into (18), the control law has the form: $T_1 \frac{d}{dt}|x_4 - \varphi_1(x_1, x_2, x_3)| + |x_4 - \varphi_1(x_1, x_2, x_3)| = 0$. From the above equation and replacing $\dot{x}_4$ from the mathematical model (6), the internal control signal is described as follows:

$$\tau = -k(x_1 - x_3) + J\sum_{i=1}^{3}\frac{\partial\varphi_1}{\partial x_i}\frac{\partial x_i}{\partial t} - \frac{J}{T_1}|x_4 - \varphi_1(x_1, x_2, x_3)|sign(s_1) \tag{19}$$

When the system enters the manifold, the performance point of the system touches the intersection of the $\psi_1 = 0$ manifold, then the system (6) will be seperated and the dynamics of the closed system are described by the (20):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{mgl}{I}sin(x_1) - \frac{k}{I}(x_1 - x_3) \\ \dot{x}_3 = \varphi_1(x_1, x_2, x_3) \end{cases} \tag{20}$$

The function $\varphi_1(x_1, x_2, x_3)$ in the decomposed system (20) can be considered as an internal control signal. In the second step of the synthesis, to find for the control law and to determine the function $\varphi_1(x_1, x_2, x_3)$, an additional invariant manifold is introduced, which will ensure the stability of the closed-loop system and the response of invariant technology (16). A second invariant manifold is chosen:

$$\psi_2 = x_3 - \varphi_2(x_1, x_2) = 0 \tag{21}$$

To ensure the internal stability of the system (20) similar to the first step, the macro variable ψ_2 must satisfy the roots of the basic equation:

$$T_2\dot{\psi}_2 + \psi_2 = 0 \tag{22}$$

where $T_2>0$ ensures the asymptotic stability of the system. Substituting (21) into (22) to get the control law:

$$T_2\frac{d}{dt}(x_3 - \varphi_2(x_1, x_2)) + (x_3 - \varphi_2(x_1, x_2)) = 0 \tag{23}$$

From the above equation and replacing $\dot{x}_3$ from the system of internal dynamic (20), the internal control signal φ_1 has the form:

$$\varphi_1 = \sum_{i=1}^{2}\frac{\partial\varphi_2}{\partial x_i}\frac{\partial x_i}{\partial t} - \frac{x_3 - \varphi_2(x_1, x_2)}{T_2} \tag{24}$$

When the system enters the manifold, the performance point of the system touches the intersection of the manifold $\psi_2 = 0$, then the dynamic system (20) is separated and the closed system are described by (25):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{mgl}{I} sin(x_1) - \frac{k}{I} x_1 + \frac{k}{I} \varphi_2(x_1, x_2) \end{cases} \quad (25)$$

In the third step of the synthesis, to find for the control and to determine the function $\varphi_2(x_1, x_2)$, a third invariant manifold is constructed, which will ensure the internal stability of the closed-loop system and the response of invariant technology (16).

$$\psi_3 = x_2 - K(x_1 - x_d) = 0 \quad (26)$$

The dynamic system (25) on the manifold in the last step (26) are rewritten as:

$$\dot{x}_1 = K(x_1 - x_d) \quad (27)$$

From the dynamic (27), the asymptotic stability condition at $x_1 = x_d$ is $K < 0$. To satisfy the condition $\psi_2 = 0$, the macro variable ψ_2 must satisfy the solution of (28):

$$T_3 \dot{\psi}_3 + \psi_3 = 0 \quad (28)$$

where $T_3 > 0$ is the condition for asymptotic stability of the system with the invariant manifold.

Substitute (26) into (28) to find the internal control signal $\varphi_2(x_1, x_2)$.

$$T_3 \frac{d}{dt}(x_2 - K(x_1 - x_d)) + x_2 - K(x_1 - x_d) = 0 \quad (29)$$

Furthermore, the equations of the decomposed system (25) are substituted into (29), resulting in the resulting (30):

$$T_3 \left(-\frac{mgl}{I} sin(x_1) - \frac{k}{I} x_1 + \frac{k}{I} \varphi_2(x_1, x_2) - K(x_2 - \dot{x}_d) \right) + x_2 - K(x_1 - x_d) = 0 \quad (30)$$

From (30) the internal control signal $\varphi_2(x_1, x_2)$ is found:

$$\varphi_2(x_1, x_2) = \frac{mgl}{k} sin(x_1) + x_1 + \frac{IK}{k}(x_2 - \dot{x}_d) - \frac{I}{kT_3}(x_2 - K(x_1 - x_d)) \quad (31)$$

From the (17), (21), (31) and invariant technology (10), the control law u for the FJM has the form.

$$\tau = -k(x_1 - x_3) + J \sum_{i=1}^{3} \frac{\partial \varphi_1}{\partial x_i} \frac{\partial x_i}{\partial t} - \frac{J}{T_1} |x_4 - \varphi_1(x_1, x_2, x_3)| sign(s_1) \quad (32)$$

where $\varphi_1 = \left(\frac{mgl}{k} cos(x_1) + 1 + \frac{IK}{kT_3} \right) x_2 + \frac{KT_3}{kT_3}(-mgl\, sin(x_1) - k(x_1 - x_3)) - \frac{x_3 - \varphi_2(x_1, x_2)}{T_2}$

Check the stability of the control law τ and the response of the condition for the sliding mode [28]. Choosing a positive Lyapunov function of the form:

$$V = 0.5\ s_1^2 \quad (33)$$

When the control law (29) affects on the system (6), the derivative of the Lyapunov function (30) has the form:

$$\dot{V} = s_1 \dot{s}_1 = s_1 \left(\dot{x}_4 - \sum_{i=1}^{3} \frac{\partial \varphi_1}{\partial x_i} \frac{\partial x_i}{\partial t} \right) = -s_1 \frac{1}{T_1} |x_4 - \varphi_1(x_1, x_2, x_3)| sign(s_1) = -\frac{1}{T_1} |s_1|^2 \leq 0 \quad (34)$$

In (34) and the control law (32) always guarantees the system (6) global stability. In addition, the sliding surface $s_1 = x_4 - \varphi_1(x_1, x_2, x_3)$ and its derivative $\dot{s}_1$ can be expressed from the functional (18). Then the condition for the appearance of sliding mode (32) is created by the choice of the parameters $T_1 > 0$.

3.3. Synthesis of SMC for flexible coupling

SMC law presented in the studies [3] is synthesized for this system to compare the results of SMC law based on serial invariant manifolds. Select the sliding surface of the controller σ has the form:

$$\sigma = \sum_{i=1}^{n-1} \alpha_i e_i + e_n \tag{35}$$

where n = 4, α_i (i = 1,2,3) are positive constants chosen to ensure the dynamics of asymptotic stability of the system on the sliding surface, the errors e_i are calculated as (36):

$$e_1 = x_1 - x_d; \quad e_2 = x_2 - \dot{x}_d; \quad e_3 = \ddot{e}_1 = f_1(x_1, x_2, x_3) - \ddot{x}_d;$$
$$e_4 = \dddot{e}_1 = \frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f_1 + \frac{\partial f_1}{\partial x_3} x_4 - \dddot{x}_d. \tag{36}$$

in (35) is written as

$$\sigma = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + e_4 \tag{37}$$

where α_i are the coefficients given such that the characteristic polynomial of (35) is a Hurwitz polynomial (all roots have negative real part).

$$\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + e_4 = 0 \tag{38}$$

Then all roots of the characteristic equation are on the left of the complex plane, so *e(t)* will approach 0 as t approaches ∞. The equation $\sigma = 0$ defines a curved surface *S* in *n*-dimensional space called the sliding surface *S*. The problem of controlling the output signal *x(t)* following the setting signal $x_d(t)$ is transformed into the problem of finding the control signal $\tau(t)$ such that $\sigma \rightarrow 0$.

Selection of Lyapunov function $V = \frac{1}{2}\sigma^2$

Derivative of Lyapunov function $V = \sigma\dot{\sigma}$

To $\sigma \rightarrow 0$ it is necessary to choose the control signal $\tau(t)$ such that $\dot{V} < 0$, from expressions (36) and (37).

$$\dot{\sigma} = \alpha_1 \dot{e}_1 + \alpha_2 \dot{e}_2 + \alpha_3 \dot{e}_3 + \dot{e}_4 = \alpha_1 x_2 + \alpha_2 f_1 + \alpha_3 \left(\frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f_1 + \frac{\partial f_1}{\partial x_3} x_4 \right)$$
$$+ \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_1} x_2\right) + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_2}\right) f_1 + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_3}\right) x_4 + \frac{\partial f_1}{\partial x_3}\left(f_2 + \frac{u}{J}\right) \tag{39}$$

Then, the the sliding control law is defined as:

$$\tau = -\left(\frac{\partial f_1}{\partial x_3}\right)^{-1} \begin{Bmatrix} \alpha_1 x_2 + \alpha_2 f_1 + \alpha_3 \left(\frac{\partial f_1}{\partial x_1} x_2 + \frac{\partial f_1}{\partial x_2} f_1 + \frac{\partial f_1}{\partial x_3} x_4 \right) + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_1} x_2\right) + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_2}\right) f_1 + \\ + \frac{d}{dt}\left(\frac{\partial f_1}{\partial x_3}\right) x_4 + \frac{\partial f_1}{\partial x_3}(f_2) - \alpha_1 \dot{x}_d - \alpha_2 \ddot{x}_d - \alpha_3 \dddot{x}_d - x_d^{(4)} + \Gamma sat(\sigma) \end{Bmatrix} \tag{40}$$

where

$$\frac{\partial f_1}{\partial x_1} = \frac{-k}{I} - \frac{mgl}{I} cos(x_1); \frac{\partial f_1}{\partial x_2} = 0; \frac{\partial f_1}{\partial x_3} = \frac{k}{I}; \tag{41}$$

in (40), the saturation function *sat(σ)* is chosen instead of the sign function *sign(σ)* to reduce chattering which can damage the actuator, the parameter Γ is a positive constant.

4. RESULTS AND DISCUSSION

4.1. Description of the intended results

In-depth simulations have been performed to demonstrate the potential of the proposed control law for the FJM system. The proposed control parameters are selected through the parameters T_i (i=1:3) and K. These parameters reflect the convergence time to the selected manifolds and the total time from initial positions to the desired point can be calculated in advance. In this study, a set of parameters will be selected with the following values: K=-31; T_1=0.2; T_2=0.2; T_3=0.3. The SMC parameters were selected based on the [3] study as follows: Γ=1040; α_1=27; α_2=27; α_3=9. The implementation of the SMC control law and the proposed control law is conducted with two cases: The first case when the initial state of the system $x(0)=[0.3; 0.5; 0.3; 0.5]^T$ is stable to the origin of coordinates x_{finish}=[0; 0; 0; 0]; the second case when initial states are $x(0)=[0.3;\ 0.5;\ 0.3;\ 0.5]^T$ and angle q_1 is tracking the desired trajectory which has the form $x_1=\sin(\omega t)$ with frequency $\omega=0.5$. The maximum torque generated by the actuator is 10 Nm.

4.2. Numerical simulation results

Figures 2(a) and (b) illustrates the angle response q_1, q_2 of the FJM system with two control laws for the first case. Both the basic SMC and the SMC based on invariant manifolds are stable about the origin of coordinates. The actual angles of the joints are rapidly approaching the desired values. Besides, it also noticed that the quality of the suggested controller is better. The setting time of q_1 to the error value 0.015 (rad) is 1.86 s and for the sliding controller normally is 1.92 s. During the rest of the time, the two controllers bring the system's state to the origin of coordinates without overshooting. The angle rotation with the ox axis of both controllers is very close to the set value. Figure 2(c) shows the moment generated by the proposed control law and the normal SMC. The switching phenomenon occurs at time 0.01s, and the moment generated by the proposed control law has no switching phenomenon and the torque is smoother. In addition, the initial moment of the proposed control law has the opposite direction to the moment that generates the normal SMC law, so the overshoot at the beginning of q_1, q_2 of the proposed control law is smaller than of the sliding control law shown in the period from 0 s to 3.1 s Figures 2(a) and (b).

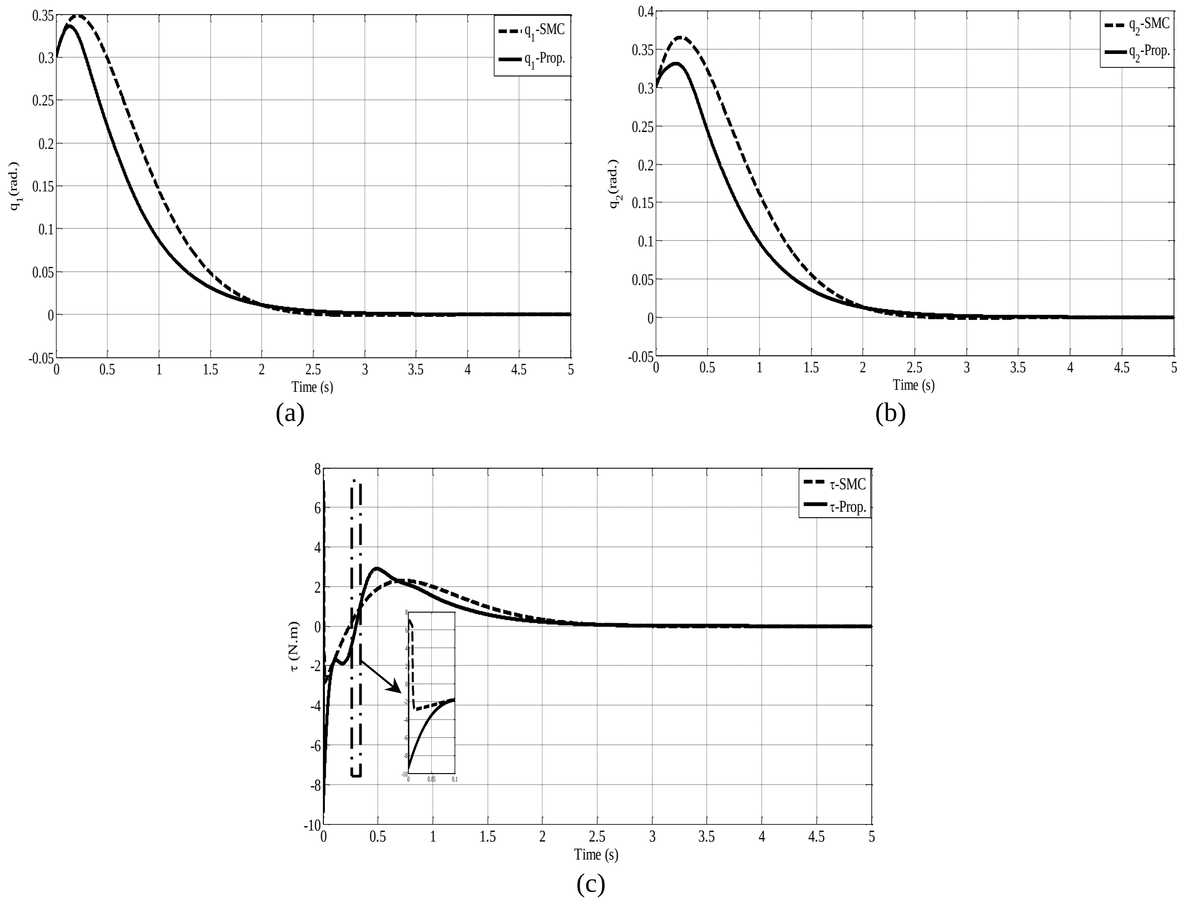

Figure 2. Simulation results when moving with non-zero initial conditions to the coordinate axis (a) response of link q_1, (b) response of link q_2, and (c) moment of motor

Figure 3 is the response of the FJM system to the above two control laws for the second case. In which Figures 3(a) and (b) is the response of link angle q_1 and tracking error when the following value has a period. The proposed control law gives better results: the time response to follow the desired trajectory for the proposed control law takes only 2.2 s while SMC takes up to 4.15 s. Besides, the large tracking error of the proposed control law is 0.019 (rad) and SMC is 0.14 (rad). In Figure 3(c) the motion of link q_2 has a smaller amplitude, which means that the link angle q_2 moves in a narrower range because the possible control of q_2 will be increased when the amplitude of the setting value increases. For the moment of the system shown in Figure 3(d), both of control laws generate similar control signals. However, at the beginning time, the moment direction of the proposed control law is opposite to that of the SMC control law and the signal function is smoother. This can be explained that in the proposed control law, it is only necessary to bring the sliding surface in the first manifold, the switching signal is smaller after entering the sliding surface. Therefore, the zero point of system will stabilize to the desired point under the effects of dynamics.

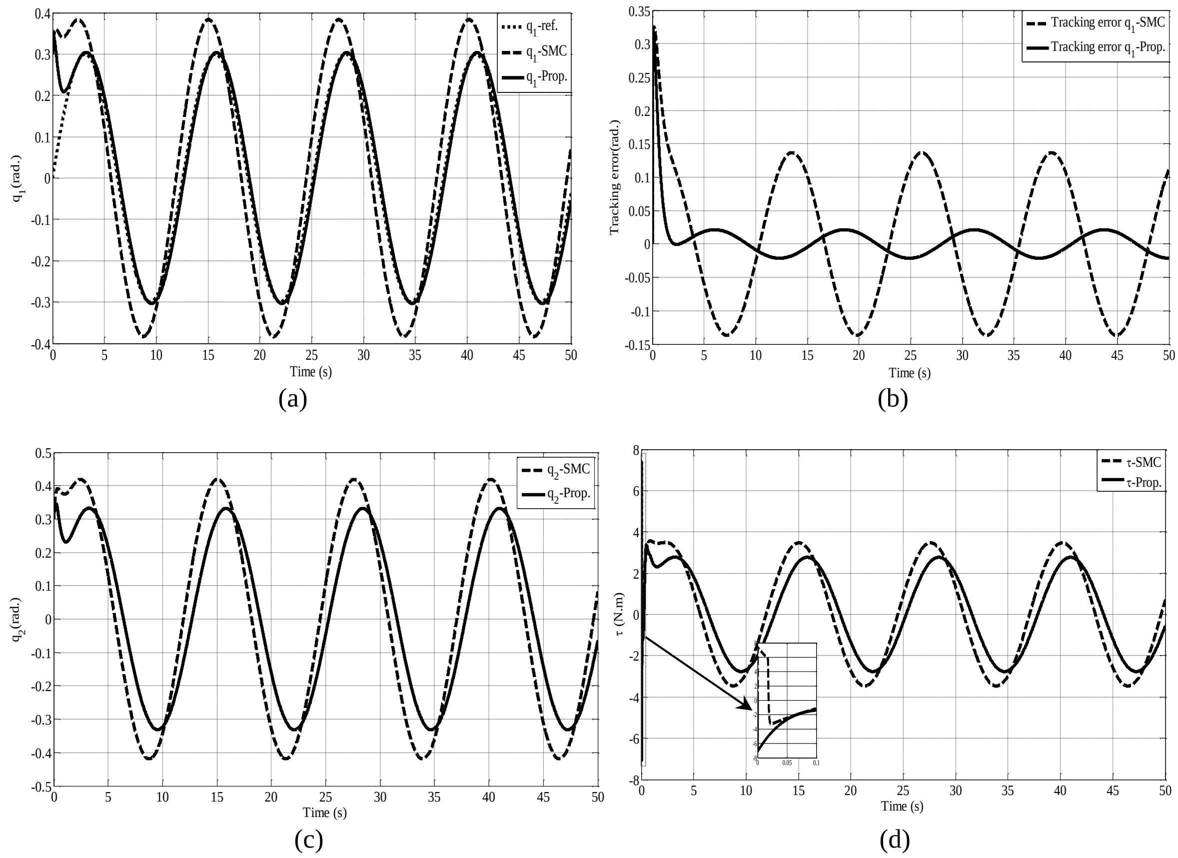

Figure 3. Simulation results when the desired trajectory is *sin (0.5t)* with non-zero initial conditions (a) response of link q_1, (b) error of tracking link q_1, (c) response of link q_2, and (d) moment of motor

5. CONCLUSION

The paper has successfully built a SMC law based on serial invariant manifolds and ADAR method. According to the numerical analysis of the results, the proposed controller shows the ability to adapt to different types of desired trajectories and good tracing ability. Furthermore, the SMC law based on these invariant manifolds gives a smooth, non-switching signal and the direction of the control law action ensures a lower overshoot than the normal SMC law.

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