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Design of unknown input observer for discrete-time Takagi Sugeno implicit systems with unmeasurable premise variables

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ABSTRACT

In this study, an unknown input observer (UIO) is developed in explicit form to estimate unmeasurable states and unknown inputs (UIs) for nonlinear implicit systems represented by the discrete-time Takagi-Sugeno implicit systems (DTSIS) in the case of unmeasurable premise variables. The method employed is based on singular value decomposition (SVD) and augmenting the state vector, which is formed partly by the system state and partly by the UIs. The convergence of the augmented state estimation error is provided by a Lyapunov function ending with solving the linear matrix inequalities (LMI). An application to a model of the rolling disc is considered to evaluate the effectiveness of the developed approach. It appears that estimated variables converge to the true variables quickly and accurately.

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1. INTRODUCTION

In state control systems, the observers of dynamic systems that are subjected to unknown inputs (Uis) are fundamental. In many industrial processes, it is impractical to measure all of the input signals of the system. In some cases, the uncertainties of particular system parameters can be described as UIs for the purpose of constructing a robust observer. An observer capable of estimating the states in the presence of UIs is required for that type of system to develop an appropriate control. Such an issue occurs in systems that are susceptible to disturbance or have inaccessible inputs, and it can present in a number of applications like fault detection and parameter identification. The main idea behind employing unknown observers for fault detection is to generate residuals from the difference between the real and estimated system outputs using an observer.

The objective of this research is to develop an unknown input observer (UIO) for nonlinear implicit systems represented by a discrete-time Takagi-Sugeno (T-S) structure with premise variables depending on unmeasurable states, where both the state and output are influenced by the UI. Note that one of the many benefits of the T-S fuzzy method is that T-S models provide an excellent technique to describe complicated nonlinear systems through the interpolation of many linear models using a membership function. Once the T-S models are obtained, the usual linear system theory may be used to analyze the complex systems. Therefore, the T-S models become powerful technical tools for modeling, design observer and control systems, as shown in [1]–[5], and the references contained therein. Otherwise, implicit systems, likewise called descriptor or singular systems are governed by a combination of differential equations and algebraic ones and have been extensively utilized in dynamic process modeling to represent the behavior of numerous industrial systems, e.g. electrical systems, robotics, and chemical processes. Some real utilizations of implicit systems may be found in [6]–[8].

The fuzzy UIO design problem for explicit or implicit T-S models attracts more and more attention and is still largely an open problem and remains an area for additional study in both continuous-time and discrete-time cases. Indeed, different advancements in the observer and its deployment to fault detection are discussed in the literature. A novel fault detection method for a robot manipulator is provided in [9], which utilizes a high-order sliding mode observer to adjust for both uncertainty and faults. A fuzzy observer design approach using T-S systems to accomplish the estimation of vehicle state taking into account of the longitudinal lateral coupling dynamics and the presence of UIs is developed in [10]. Developing an UIO for T-S systems by using an expanded state vector is the aim of the work described in [11]. Bouassem *et al*. [12] suggested another technique based on the separation between static and dynamic relations in the T-S implicit system to estimate both the system state and the UIs concomitantly. This approach allowed the estimation of both non-measurable states and unknown faults in the actuators and sensors for a class of continuous-time T-S implicit models in [13]. Louzimi *et al*. [14], a design of an UIO is proposed for a class of continuous-time T-S implicit systems satisfying the Lipschitz conditions. Similarly, in the case of discrete-time T-S models, various publications published for explicit and implicit structures such as [15]–[20]. Essabre *et al*. [21] suggest a technique for constructing an observer for a class of T-S implicit systems subject to UIs utilizing measurable premise variables that are dependent on the measurable states, input, output, or external parameters. Yoneyama [22], it is confirmed that even if the output of the system is selected as a premise variable and this output is directly impacted by perturbations (which seems to be the case in majority of realistic conditions), the T-S system acquired does not represent an exactly nonlinear system. For this reason, using the system state as a premise variable makes it possible to depict a broader class of nonlinear systems.

Motivated by the above discussions, the purpose of our study is not to compare our technique to previous ones, but rather to generalize our results from the case of T-S implicit models with measurable premise variables [21] to the case of T-S implicit models with unmeasurable premise variables. Using the singular value decomposition technique, the fundamental contribution of this research is to develop a UI observer design for a class of discrete-time Takagi-Sugeno implicit systems (DTSISs) models with unmeasurable premise variables, allowing for the estimate of UIs and unknown states at the same time. In comparison to T-S models that employ just measurable premise variables, the T-S model using unmeasurable premise variables is an important structure because it can represent a wider variety of nonlinear systems and can describe precisely the behaviour of the general nonlinear system. The convergence of the state estimation error is established by using the Lyapunov theory, and the stability criteria are expressed as linear matrix inequalities (LMIs). The suggested result in this research is presented in an explicit form.

The paper is organized as follows: the mathematics of the DTSISs subject to UI to be researched is explained in section 2. Section 3 introduces the major result of this work. Finally, this theoretical result is applied to a model of the rolling disc to show the excellent performance of the obtained result.

2. DESCRIPTION OF THE CONSIDERED MODEL

The model of the DTSIS that we study in this article is of the following form:

$$\begin{cases} Mx_{k+1} = \sum_{i=1}^{q} \varphi_i(\xi_k)(A_i x_k + B_i u_k + C_i d_k) \\ y_k = Dx_k + Ed_k \end{cases} \tag{1}$$

where $x_k \in R^n$: the states vector, $u_k \in R^m$: the control input, $y_k \in R^p$: the measured output vector, $d_k \in R^r$: the unknown input $M \in R^{n\times n}$. The matrices such that rank(M)<n, $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $C_i \in R^{n\times r}$, $D \in R^{p\times n}$ and $E \in R^{p\times r}$ are real known constant matrices.

The premise variable is included in the vector ξ_k. $\varphi_i(\xi_k)$ are the weighting functions that guarantee the transition between the contributions of each sub-model:

$$\begin{cases} Mx_{k+1} = A_i x_k + B_i u_k + C_i d_k \\ y_k = Dx_k + Ed_k \end{cases} \tag{2}$$

they hold the convex su property:

$$\sum_{i=1}^{q} \varphi_i(\xi_k) = 1;\ 0 \le \varphi_i(\xi_k) \le 1\ ; \tag{3}$$

$i = 1, \dots, q$

we consider in this study that the weighting functions $\varphi_i(\xi_k)$ are dependent on unmeasurable premise variables (system state).

2.1. Assumption 1

Assume d is taken as a constant in each interval, i.e.:

$$d_{k+1} = d_k;\ k \in [T_1\ T_2];\ \forall\, T_1, T_2 \in R^+ \tag{4}$$

to concomitanly estimate the UIs and system state for system (1), we use the augmented system structure, defined as follows:

$$\begin{cases} H\eta_{k+1} = \sum_{i=1}^{q} \varphi_i(\xi_k)\,(Q_i\eta_k + S_i u_k) \\ y_k = F\eta_k \end{cases} \tag{5}$$

where:

$$\begin{cases} \eta_k = \begin{pmatrix} x_k \\ d_k \end{pmatrix} \\ H = \begin{pmatrix} M & 0 \\ 0 & I \end{pmatrix} \\ Q_i = \begin{pmatrix} A_i & C_i \\ 0 & I \end{pmatrix} \\ S_i = \begin{pmatrix} B_i \\ 0 \end{pmatrix} \\ F = (D\ \ E) \end{cases} \tag{6}$$

Before we get to the main point, consider the subsequent assumption.

2.2. Assumption 2

Suppose that: i) H1) (H, Q_i) are regular, i.e. $det(zH - Q_i) \neq 0\ \forall z \in C$, ii) H2) All sub models (2) are impulse observable and detectable, and iii) H3) $rank\begin{pmatrix} H \\ F \end{pmatrix} = \sigma = n + r$.

Taking into account the hypothesis H3), there exists a non-singular matrix $\begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix}$ such that:

$$\begin{cases} J_1 H + J_2 F = I \\ J_3 H + J_4 F = 0 \end{cases} \tag{7}$$

where $J_1 \in R^{\sigma\times\sigma}$, $J_2 \in R^{\sigma\times p}$, $J_3 \in R^{p\times p}$ and $J_4 \in R^{p\times p}$ are constant matrices determined from the singular value decomposition of $\begin{pmatrix} H \\ F \end{pmatrix}$.

3. MAIN RESULT: FUZZY UNKOWN INPUTS OBSERVER DESIGN MAI

The goal of this paragraph is to provide a novel observer design approach to estimate the UIs and the unmeasurable states of DTSIS (1). To do this, the DTSIS (1) is converted into the equivalent system (5). Thus, the suggested fuzzy observer has the following structure:

$$\begin{cases} \mu_{k+1} = \sum_{i=1}^{q} \varphi_i(\hat{\xi}_k)\,(N_i\mu_k + L_{1i}u_k + L_{2i}y_k + G_i u_k) \\ \hat{\eta}_k = \mu_k + J_2 y_k + K J_4 y_k \end{cases} \tag{8}$$

Where $\hat{\eta}_k$: the estimated augmented state vector; $\hat{\xi}_k$: the estimate of the premise vector, the gains matrices N_i, L_{1i}, L_{2i} and $G_i$must be calculated such that $\hat{\eta}_k$ converges to η_k exponentially. Let us develop the augmented state estimation error in order to set the conditions for the exponential convergence of the observer (8):

$$\varepsilon_k = \eta_k - \hat{\eta}_k \tag{9}$$

by substituting (5), (7) and (8) into (9) we obtain:

$$\varepsilon_k = (KJ_1 + KJ_3 y_k)H\eta_k - \mu_k \tag{10}$$

from (5) and (8) it follows that the dynamics of this error is:

$$\varepsilon_{k+1} = \sum_{i=1}^{q} \varphi_i(\xi_k)(J_1 + KJ_3)\,(Q_i\eta_k + S_iu_k) - \sum_{i=1}^{q} \varphi_i\big(\hat{\xi}_k\big)\,(N_i\mu_k + L_{1i}u_k + L_{2i}y_k + G_iu_k) \quad (11)$$

using (10), (11) can be written as:

$$\varepsilon_{k+1} = \sum_{i=1}^{q} \varphi_i(\xi_k)(J_1 + KJ_3)\,(Q_i\eta_k + S_iu_k) + \sum_{i=1}^{q} \varphi_i\big(\hat{\xi}_k\big)\,N_i\varepsilon_k - \sum_{i=1}^{q} \varphi_i\big(\hat{\xi}_k\big)\,((N_i(J_1 + KJ_3)H + L_{1i}F + L_{2i}F)\eta_k + G_iu_k) \quad (12)$$

provided the matrices N_i, L_{1i}, L_{2i} and G_i and K satisfy:

$$N_i(J_1 + KJ_3)H + L_{1i}F + L_{2i}F = (J_1 + KJ_3)Q_i \quad (13)$$

$$G_i = (J_1 + KJ_3)S_i \quad (14)$$

then, using (7) and (13), we get:

$$N_i = (J_1 + KJ_3)Q_i - L_{2i}F + (N_1(J_2 + KJ_4) - L_{1i})F \quad (15)$$

take:

$$L_{1i} = N_i(J_2 + KJ_4) \quad (16)$$

then:

$$N_i = (J_1 + KJ_3)Q_i - L_{2i}F \quad (17)$$

it follows system (12) is equivalent to:

$$\varepsilon_{k+1} = \sum_{i=1}^{q} \varphi_i\big(\hat{\xi}_k\big)N_i\varepsilon_k + \sum_{i=1}^{q}(\varphi_i(\xi_k) - \varphi_i\big(\hat{\xi}_k\big))(J_1 + KJ_3)(Q_i\eta_k + S_iu_k) \quad (18)$$

note that:

$$\sum_{i=1}^{q}(\varphi_i(\xi_k) - \varphi_i(\hat{\xi}_k))Q_i = \sum_{i,j=1}^{q} \varphi_i(\xi_k)\varphi_j(\hat{\xi}_k)(Q_i - Q_j) \quad (19)$$

$$\sum_{i=1}^{q}(\varphi_i(\xi_k) - \varphi_i(\hat{\xi}_k))S_i = \sum_{i,j=1}^{q} \varphi_i(\xi_k)\varphi_j(\hat{\xi}_k)(S_i - S_j)$$

the (18) is then transformed as (20):

$$\varphi_j\big(\hat{\xi}_k\big))(J_1 + KJ_3)\big(\Delta Q_{ij}\eta_k + \Delta S_{ij}u_k\big) \quad (20)$$

where $\Delta Q_{ij} = Q_i - Q_j$ and $\Delta S_{ij} = S_i - S_j$. By multiplying by $\sum_{i,j=1}^{q}(\varphi_i(\xi_k)$, (20) can be simplified to the (21):

$$\varepsilon_{k+1} = \sum_{i,j=1}^{q} \varphi_i(\xi_k)\varphi_i\big(\hat{\xi}_k\big)(N_j\varepsilon_k + \Phi_{ij}\eta_k + \Gamma_{ij}u_k) \quad (21)$$

where

$$\begin{cases} \Phi_{ij} = (J_1 + KJ_3)\Delta Q_{ij} \\ \Gamma_{ij} = (J_1 + KJ_3)\Delta S_{ij} \\ i,j \in (1, \dots, q) \end{cases} \quad (22)$$

considering $\tilde{\varepsilon}_k = (\varepsilon_k^T \ \eta_k^T)^T$, we get:

$$\tilde{H}\tilde{\varepsilon}_{k+1} = \sum_{i,j=1}^{q} \varphi_i(\xi_k)\varphi_j(\hat{\xi}_k)(\tilde{Q}_{ij}\tilde{\varepsilon}_k + \tilde{S}_{ij}u_k) \tag{23}$$

where

$$\begin{cases} \tilde{H} = \begin{pmatrix} I & 0 \\ 0 & H \end{pmatrix} \\ \tilde{Q}_{ij} = \begin{pmatrix} N_j & \Phi_{ij} \\ 0 & Q_i \end{pmatrix} \\ \tilde{S}_{ij} = \begin{pmatrix} \Gamma_{ij} \\ S_i \end{pmatrix} \end{cases} \tag{24}$$

To introduce the convergence conditions of the suggested observer (8), we form the following theorem. Theorem: according to hypotheses 1 and 2, if for DTSIS (1) there exist symmetric matrices P_1, P_2, Q, and the matrices W_j, $j \in (1, \dots, q)$ satisfying the LMI (25), then the observer gains that guarantee the convergence to zero of the estimation error can be determined:

$$\begin{pmatrix} m_{11} & m_{21}^T & m_{31}^T & m_{41}^T \\ m_{21} & m_{22} & m_{32}^T & m_{42}^T \\ m_{31} & m_{32} & m_{33} & m_{43}^T \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} < 0 \tag{25}$$

$i, j, \in (1, \dots, q)$

where

$$\begin{cases} m_{11} = -P_1 \\ m_{22} = -Q_i^T P_2 Q_i - H^T P_2 H \\ m_{33} = S_i^T P_2 S_i \\ m_{44} = -P_1 \\ m_{21} = 0 \\ m_{31} = 0 \\ m_{41} = P_1 J_1 Q_i + Q J_3 Q_{j-} W_j F \\ m_{42} = P_1 J_1 \Delta Q_{ij} + Q J_3 \Delta Q_{ij} \\ m_{32} = S_i^T P_2 Q_i \\ m_{43} = 0 \end{cases} \tag{26}$$

with J_1 , J_2 , $J_{3,}$ and J_4 are such that (7) is satisfied.

The observer gains N_j, L_{1j}, and G_j are given such that (14), (16), and (17) are satisfied. K and L_{2j} are obtained by:

$$\begin{cases} K = P_1^{-1} Q \\ L_{2j} = P_1^{-1} W_{j_1} \end{cases} \tag{27}$$

Proof: the following quadratic Lyapunov function is used to demonstrate the theorem:

$$V_k = \tilde{\varepsilon}_k{}^T \tilde{H}^T P \tilde{H} \tilde{\varepsilon}_k,\ \tilde{H}^T P \tilde{H} \geq 0,\ P = P^T > 0 \tag{28}$$

with:

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \tag{29}$$

its variation $\Delta V = V_{k+1} - V_k$ along the error dynamics (23) is calculated by:

$$\Delta V = \tilde{\varepsilon}_{k+1}{}^T \tilde{H}^T P \tilde{H} \tilde{\varepsilon}_{k+1} - \tilde{\varepsilon}_k{}^T \tilde{H}^T P \tilde{H} \tilde{\varepsilon}_k \tag{30}$$

by using (23), (29) becomes as (31):

$$\Delta V = \sum_{i,j=1}^{q} \varphi_i(\xi_k)\varphi_j(\hat{\xi}_k)\,(\tilde{Q}_{ij}\tilde{\varepsilon}_k + \tilde{S}_{ij}u_k)^T P(\tilde{Q}_{ij}\tilde{\varepsilon}_k + \tilde{S}_{ij}u_k) - \tilde{\varepsilon}_k{}^T\,\tilde{H}^T P\tilde{H}\tilde{\varepsilon}_k \quad (31)$$

multiplying by $\sum_{i,j=1}^{q} \varphi_i(\xi_k)\varphi_j(\hat{\xi}_k)$, (30) can be reduced to the (32):

$$\Delta V = \sum_{i,j=1}^{q} \varphi_i(\xi_k)\varphi_j(\hat{\xi}_k)\,[\tilde{\varepsilon}_k{}^T \quad u_k^T]\,\Omega_{ij}\begin{bmatrix}\tilde{\varepsilon}_k\\ u_k\end{bmatrix} \quad (32)$$

where:

$$\Omega_{ij} = \begin{pmatrix}\tilde{Q}_{ij}^T P\tilde{Q}_{ij} - \tilde{H}^T P\tilde{H} & \tilde{Q}_{ij}^T P S_{ij}\\ S_{ij}^T P\tilde{Q}_{ij} & S_{ij}^T P S_{ij}\end{pmatrix} \quad (33)$$

$i, j, \in (1, \dots, q)$

thus, the convergence of the estimation error is ensured if the following condition is guaranteed:

$$\Omega_{ij} < 0 \quad (34)$$

from (6), and (24), (34) can be written as:

$$\begin{pmatrix}\theta_{11} & \theta_{12} & \theta_{13}\\ \theta_{21} & \theta_{22} & \theta_{23}\\ \theta_{31} & \theta_{32} & \theta_{33}\end{pmatrix} < 0 \quad (35)$$

$i, j, \in (1, \dots, q)$

$$\begin{cases}\theta_{11} = N_j^T P_1 N_j - P_1\\ \theta_{21} = \Phi_{ij}^T P_1 N_j\\ \theta_{31} = \Gamma_{ij}^T P_1 M_j\\ \theta_{12} = N_j^T P_1 \Phi_{ij}^T\\ \theta_{22} = \Phi_{ij}^T P_1 \Phi_{ij} + Q_i^T P_2 Q_i - H^T P_2 H\\ \theta_{32} = \Gamma_{ij}^T P_1 \Phi_{ij} + S_i^T P_2 Q_i\\ \theta_{13} = N_j^T P_1 \Gamma_{ij}\\ \theta_{23} = \Phi_{ij}^T P_1 \Gamma_{ij} + Q_i P_2 S_i\\ \theta_{33} = \Gamma_{ij}^T P_1 \Gamma_{ij} + S_i^T P_2 S_i\end{cases} \quad (36)$$

based on the schur complement [23], (35) is equivalent to the following conditions:

$$\begin{pmatrix}-P_1 & 0 & 0 & N_j^T P_1\\ 0 & Q_i^T P_2 Q_i - H^T P_2 H & Q_i^T P_2 S_i & \Phi_{ij}^T P_1\\ 0 & S_i^T P_2 Q_i & S_i^T P_2 S_i & \Gamma_{ij}^T P_1\\ P_4 N_j & P_1 \Phi_{ij} & P_1 \Gamma_{ij} & -P_1\end{pmatrix} < 0 \quad (37)$$

$i, j, \in (1, \dots, q)$

then, from (17), and (22), and use of the change of variable:

$$\begin{cases}Q = P_1 K\\ W_j = P_1 L_{2j}\end{cases} \quad (38)$$

the LMIs conditions (25) are obtained. The proof is completed.

4. NUMERICAL APPLICATION TO ROLLING DISC

To evaluate the suggested observer, we consider the following DTSIS of the rolling process given in [24] which is assumed to be affected by an unknown input variable as follows:

$$\begin{cases} Mx_{k+1} = \sum_{i=1}^{2} \varphi_i(\xi_k)(A_i x_k + B_i u_k + C d_k) \\ y_k = D x_k + E d_k \end{cases} \quad (39)$$

where $x_k = [x_{1k}, \dots, x_{4k}]$: the state vector, x_{1k}: the position of the centre of the disc, x_{2k}: the translational velocity of the centre of the disc, x_{3k}: the angular velocity of the disc, x_{4k}: the contact force, u_k and y_k: the control and the output measurements vectors respectively. d_k: the unknown input. The numerical matrices values are:

$$A_1 = \begin{pmatrix} 1 & 0.01 & 0 & 0 \\ -0.025 & 0.9925 & 0 & 0.0003 \\ 0 & 0.01 & -0.004 & 0 \\ -0.025 & -0.0075 & 0 & 0.0008 \end{pmatrix}, A_2 = \begin{pmatrix} 1 & 0.01 & 0 & 0 \\ -0.027 & 0.9925 & 0 & 0.0003 \\ 0 & 0.01 & -0.004 & 0 \\ -0.025 & -0.0075 & 0 & 0.0008 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -0.0013 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \quad M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 \\ 0.01 \\ 0 \\ 0 \end{pmatrix}$$

The weighting functions are given by: $\varphi_1(\xi_k) = 1 - 12.7551 x_{1k}^2$ and $\varphi_2(\xi_k) = 12.7551 x_{1k}^2$. Note that to apply the suggested UIO (8) for (39), just rewrite the model (39) in its equivalent form (5) as indicated in section 2. Therefore, under hypothesis H3) the following matrices J_1, J_2, J_3 and J_4 verifying (7) have been obtained:

$$J_1 = \begin{pmatrix} 0.8 & 0.2 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0 & 0 & 0 \\ -0.2 & 0.2 & 0 & 0 & 0 \\ -0.4 & -0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0.2 & -0.2 & -0.2 \\ -0.2 & 0.2 & 0.2 \\ 0.2 & 0.8 & -0.2 \\ 0.4 & -0.4 & 0.6 \\ 0 & 0 & 0 \end{pmatrix}$$

$$J_3 = \begin{pmatrix} -0.2828 & 0.2828 & -0.5774 & -0.5164 & 0 \\ 0.2 & -0.2 & -0.8165 & 0.3651 & 0 \\ 0.2828 & -0.2828 & 0 & -0.7746 & 0 \end{pmatrix}$$

$$J_4 = \begin{pmatrix} 0.2828 & -0.2828 & -0.2828 \\ -0.2 & 0.2 & 0.2 \\ -0.2828 & 0.2828 & 0.2828 \end{pmatrix}$$

the following observer gains result from the solving of LMIs (25) of the Theorem:

$$N_1 = \begin{pmatrix} 0.0355 & 0.0355 & 0 & -0.0355 & 0.0265 \\ 0.0355 & 0.0354 & 0 & -0.0354 & 0.0269 \\ 0 & 0 & 0 & 0 & 0.0002 \\ -0.0355 & -0.0355 & 0 & 0.0355 & -0.0269 \\ 0.0258 & 0.0232 & -0.0009 & -0.024 & 0.3628 \end{pmatrix}$$

$$N_2 = \begin{pmatrix} 0.0355 & 0.0355 & 0 & -0.0355 & 0.0265 \\ 0.0355 & 0.0354 & 0 & -0.0354 & 0.0269 \\ 0 & 0 & 0 & 0 & 0.0002 \\ -0.0355 & -0.0355 & 0 & 0.0355 & -0.0269 \\ 0.0258 & 0.0232 & -0.0009 & -0.024 & 0.3628 \end{pmatrix}$$

$$L_{11} = \begin{pmatrix} -1.4729 & 1.4729 & 1.4374 \\ -1.5011 & 1.5011 & 1.4657 \\ -0.0155 & 0.0155 & 0.0155 \\ 1.5020 & -1.5019 & -1.4665 \\ -22.9698 & 22.9689 & 22.9457 \end{pmatrix}, L_{12} = \begin{pmatrix} -1.4729 & 1.4729 & 1.4374 \\ -1.5011 & 1.5011 & 1.4657 \\ -0.0155 & 0.0155 & 0.0155 \\ 1.5020 & -1.5019 & -1.4665 \\ -22.9698 & 22.9689 & 22.9457 \end{pmatrix}$$

$$L_{21}=\begin{pmatrix}-1.6832 & 2.0345 & 1.7173\\ -1.727 & 2.0782 & 1.7611\\ -0.0242 & 0.0241 & 0.0242\\ 1.7285 & -2.0797 & -1.7626\\ 63.6925 & -63.4787 & -63.6366\end{pmatrix},\quad L_{22}=\begin{pmatrix}-1.6832 & 2.0345 & 1.7173\\ -1.727 & 2.0782 & 1.7611\\ -0.0242 & 0.0241 & 0.0242\\ 1.7285 & -2.0797 & -1.7626\\ 63.6925 & -63.4787 & -63.6366\end{pmatrix}$$

$$G_1=\begin{pmatrix}0.0033\\ 0.0034\\ 0\\ -0.0034\\ -0.0796\end{pmatrix},\quad G_2=\begin{pmatrix}0.0033\\ 0.0034\\ 0\\ -0.0034\\ -0.796\end{pmatrix},\quad K=\begin{pmatrix}55.5406 & 68.3030 & -1.4106\\ 54.7603 & 68.8198 & -0.5902\\ -0.2466 & 0.1520 & 0.2673\\ -55.0423 & -68.6124 & 0.8740\\ -92.2846 & 130.4449 & 40.7553\end{pmatrix}$$

The UI, which is applied during the interval [2s, 4s], is shown in Figure 1.

The simulation results shown in Figures 1-4 illustrate that the presence of the UI causes an undesirable and noticeable variation in system states at the moment of t the UI appears. Consistent with the results from Figures 1-5, we can see that the suggested observer developed by employing these numerical gain values N_i, L_{1i}, L_{2i} and G_i permits the estimated states to converge to the actual state as rapidly as possible, restoring desired performance and ensuring stability despite the existence of UI.

To show the advantage and justify the effectiveness of the suggested method, we compare the proposed estimator with other advanced methods. For example, the technique described in [25] is founded totally on the separation between the differential equations and algebraic ones inside the DTSIS that is applicable only for systems that satisfy the criteria of rank specified in the study. Another benefit of our technique over others that employ DTSIS with premise variables satisfying Lipschitz criteria (see [14]) is the absence of the Lipschitz assumption of the weighting functions, that allowed us to reduce the LMI constraints.

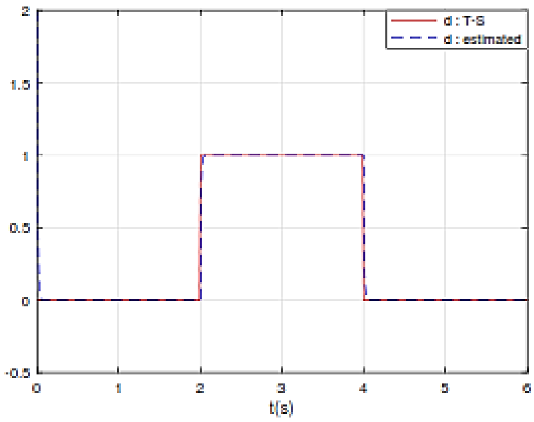

Figure 1. Unknown input d and its estimate

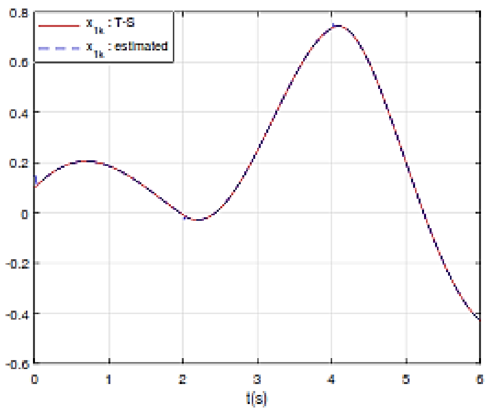

Figure 2. State variable x_1 and its estimate

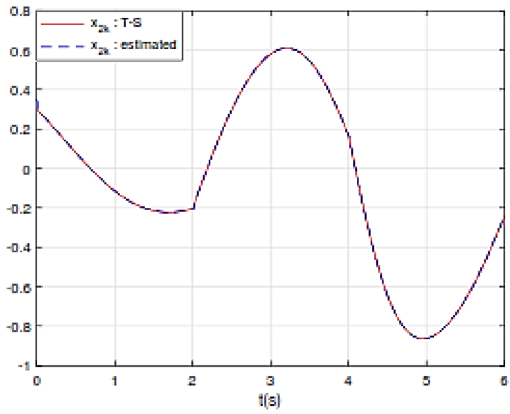

Figure 3. State variable x_2 and its estimate

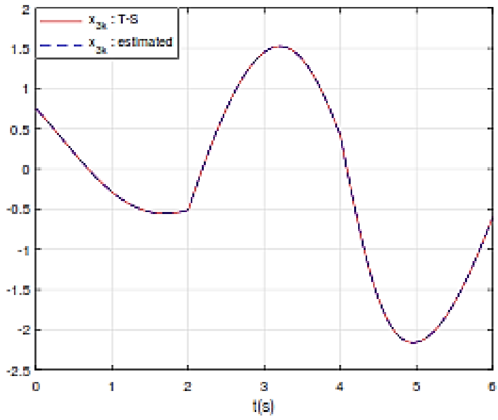

Figure 4. State variable x_3 and its estimate

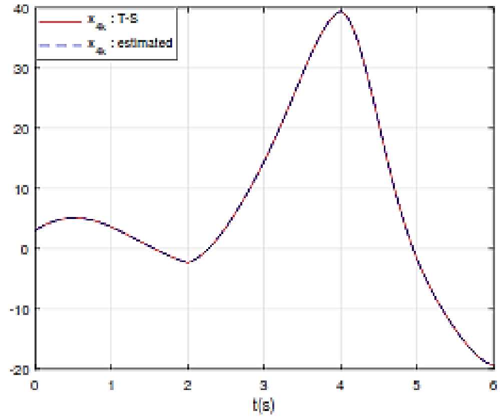

Figure 5. State variable x_4 and its estimate

5. CONCLUSION

In this research, a new UIO design method for DTSIS with unmeasurable premise variables is presented. The approach is based on the singular value decomposition and the utilization of an augmented system composed of the state variables of the model and UIs. The suggested solution allowed for the estimation of both the system status and the UIs at the same time. The Lyapunov theory is used to investigate the convergence of the state estimation error, and the presence of the condition assuring this convergence is represented in terms of LMI. Finally, an example of application to a rolling disc is presented to verify the performance of the suggested technique in estimating the states and the UI.

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