

On picture fuzzy ideals on commutative rings

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ABSTRACT

In this paper, we focus on combining the theories of picture fuzzy sets on rings and establishing a new framework for picture fuzzy sets on commutative rings. The aim of this manuscript is to apply picture fuzzy set for dealing with several kinds of theories in commutative rings. Moreover, we introduce the notions of picture fuzzy ideals on commutative rings and some properties of them are obtained. Finally, we give suitable definitions of the operations of picture fuzzy ideals over a commutative ring, as composition, product and intersection.

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1. INTRODUCTION

The notion of L-fuzzy ideal of a ring is introduced in [1], which is related to the notion of fuzzy ideals over a ring in [2]. In 2008, Kazanci and Davvaz [3] applied the concept of fuzzy prime (primary) ideals to the theory of rough prime (primary) ideals. In 2012, Navarro *et al.* [4] introduced and studied the concept of prime fuzzy ideals over a non commutative ring. In 2016, Darani and Ghasemi [5] defined L-fuzzy 2-absorbing ideals of a commutative ring. In 2017, Sonmez *et al.* [6] introduced the notion of 2-absorbing fuzzy ideals and 2-absorbing primary fuzzy ideals of a commutative ring. In 2019, Yiarayong [7] gave a complete characterization of fuzzy quasi-prime and weakly fuzzy quasi-prime ideals. In 2020, Asif *et al.* [8] explored the concept of picture fuzzy near-rings (PFNRs) and picture fuzzy ideals (PFIs) of a near-ring (NR). In 2021, Ali and Mohammed [9] introduced and examined the notion of hesitant fuzzy and hesitant fuzzy prime ideals of a ring. Fuzzy sets of a ring have been studied in various algebraic structures, see [10]-[29].

Now in this paper we introduced and study picture fuzzy sets as generalization of a commutative ring as well as fuzzy sets. We introduce the notions of picture fuzzy ideals on commutative rings and some properties of them are obtained. Finally, we give suitable definitions of the operations of picture fuzzy ideals over a commutative ring, as composition, product and intersection.

2. PICTURE FUZZY IDEALS

In this section, we concentrate our study on the picture fuzzy ideals and investigate their fundamental properties.

Let $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$ be any picture fuzzy sets over a commutative ring R . Then A is called a **subset** of B denoted by $A \subseteq B$ if $\mu_A \leq \mu_B, \eta_A \geq \eta_B$ and $\nu_A \geq \nu_B$.

Definition 2.1 Let $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$ be any picture fuzzy sets over a commutative ring R .

1. The **intersection** of two picture fuzzy sets A and B is defined as the picture fuzzy set

$$A \cap B = (\mu_A \wedge \mu_B, \eta_A \vee \eta_B, \nu_A \vee \nu_B).$$

2. The **union** of two picture fuzzy sets A and B is defined as the picture fuzzy set

$$A \cup B = (\mu_A \vee \mu_B, \eta_A \wedge \eta_B, \nu_A \wedge \nu_B).$$

We now consider another generalized fuzzy ideal which is called a picture fuzzy ideal of a commutative ring R .

Definition 2.2 A picture fuzzy set $A = (\mu_A, \eta_A, \nu_A)$ of a commutative ring R is called a **picture fuzzy ideal** of R if

1. $A(xy) \supseteq A(x) \cup A(y)$ for all $x, y \in R$,
2. $A(x - y) \supseteq A(x) \cap A(y)$ for all $x, y \in R$.

Remark 2.3 Condition (2) of the above definition is equivalent to $A(x + y) \supseteq A(x) \cap A(y)$ and $A(-x) = A(x)$ for all $x, y \in R$.

We now present the following example satisfying above definition.

Example 2.4 Let $R = \mathbb{Z}_6$. Define the picture fuzzy set $A = (\mu_A, \eta_A, \nu_A)$ as follows:

	μ_A	η_A	ν_A
0	0.8	0.1	0.1
1	0.2	0.4	0.4
2	0.3	0.2	0.2
3	0.2	0.4	0.4
4	0.3	0.2	0.2
5	0.2	0.4	0.4

Then, clearly $A = (\mu_A, \eta_A, \nu_A)$ is a picture fuzzy ideal over a commutative ring R .

Let $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$ be any picture fuzzy sets over a commutative ring R . Define $A \oplus B = (\mu_A \oplus \mu_B, \eta_A \otimes \eta_B, \nu_A \otimes \nu_B)$ is defined by

$$(\mu_A \oplus \mu_B)(x) = \begin{cases} \bigcup_{x=y+z} \mu_A(y) \wedge \mu_B(z) & ; \exists y, z \in S, \text{ such that } x = y + z \\ 0 & ; \text{otherwise,} \end{cases}$$

$$(\eta_A \otimes \eta_B)(x) = \begin{cases} \bigcap_{x=y+z} \eta_A(y) \vee \eta_B(z) & ; \exists y, z \in S, \text{ such that } x = y + z \\ 1 & ; \text{otherwise,} \end{cases}$$

and

$$(\nu_A \otimes \nu_B)(x) = \begin{cases} \bigcap_{x=y+z} \nu_A(y) \vee \nu_B(z) & ; \exists y, z \in S, \text{ such that } x = y + z \\ 1 & ; \text{otherwise.} \end{cases}$$

Theorem 2.5 Let $A = (\mu_A, \eta_A, \nu_A), B = (\mu_B, \eta_B, \nu_B)$ and $C = (\mu_C, \eta_C, \nu_C)$ be any picture fuzzy ideals over a commutative ring R . Then the following properties hold.

1. $A(x) \leq A(0)$ for all $x \in R$.
2. $A \oplus A = A$.
3. $A \oplus A = B \oplus A$.
4. $(A \oplus B) \oplus C = A \oplus (B \oplus C)$.
5. $A \oplus 0 = A$ where $0 = (0^+, 0^-, 0^-)$ is a picture fuzzy set over R , defined by,

$$0(x) = \begin{cases} (1, 0, 0); & x = 0 \\ (0, 0, 1); & x \neq 0. \end{cases}$$

6. If $A \subseteq B$, then $A \oplus C \subseteq B \oplus C$.

- Proof.** 1. Let x be an element of R . Then we have $\mu_A(0) = \mu_A(x - x) \geq \mu_A(x) \wedge \mu_A(x) = \mu_A(x)$ and $\eta_A(0) = \eta_A(x - x) \leq \eta_A(x) \vee \eta_A(x) = \eta_A(x)$. Similarly, we check that $\nu_A(0) \leq \nu_A(x)$. Therefore $A(x) \leq A(0)$ for all $x \in R$.
2. Let a be an element of R . By (1), we have $(\mu_A \oplus \mu_A)(a) = \bigcup_{a=x+y} \mu_A(x) \wedge \mu_A(y) \geq \mu_A(a) \wedge \mu_A(0) = \mu_A(a)$ and $(\eta_A \otimes \eta_A)(a) = \bigcap_{a=x+y} \eta_A(x) \vee \eta_A(y) \leq \eta_A(a)$. Similarly, we check that $(\nu_A \otimes \nu_A)(a) \leq \nu_A(a)$. Therefore $A \subseteq A\theta A$. On the other hand, let a be an element of R . Then $(\mu_A \oplus \mu_A)(a) = \bigcup_{a=x+y} \mu_A(x) \wedge \mu_A(y) = \bigcup_{a=x+y} \mu_A(x) \wedge \mu_A(-y) \leq \bigcup_{a=x+y} \mu_A(x+y) = \mu_A(a)$ and $(\eta_A \otimes \eta_A)(a) = \bigcap_{a=x+y} \eta_A(x) \vee \eta_A(y) = \bigcap_{a=x+y} \eta_A(x) \vee \eta_A(-y) \leq \bigcap_{a=x+y} \eta_A(x+y) = \eta_A(a)$. It can be similarly proved that $(\nu_A \otimes \nu_A)(a) \leq \nu_A(a)$. Therefore $A\theta A = A$.
- 3-4. The proof is easy.
5. Let x be an element of R . Then we have
- $$\begin{aligned} (0^+ \oplus \mu_A)(a) &= \bigcup_{a=x+y} 0^+(x) \wedge \mu_A(y) \\ &\leq \bigcup_{a=0+a} 0^+(0) \wedge \mu_A(a) \\ &= \bigcup_{a=0+a} 1 \wedge \mu_A(a) \\ &= \bigcup_{a=0+a} \mu_A(a) \\ &= \mu_A(a) \end{aligned}$$
- and $(0^- \otimes \eta_A)(a) = \bigcap_{a=x+y} 0^-(x) \vee \eta_A(y) \geq \eta_A(a)$. Similarly we can prove $(0^- \otimes \nu_A)(a) \geq \nu_A(a)$. Therefore $A\theta 0 = A$.
6. The proof is easy.

3. OPERATIONS ON PICTURE FUZZY IDEALS

In this section, we give suitable definitions of the operations of picture fuzzy ideals over a commutative ring R , as composition, product and intersection. Moreover, we obtain basic properties of such picture fuzzy ideals.

Let $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$ be any picture fuzzy sets over a ring R . Define the **composition** $A \odot B = (\mu_A \circ \mu_B, \eta_A \bullet \eta_B, \nu_A \bullet \nu_B)$ and **product** $AB = (\mu_A * \mu_B, \eta_A \eta_B, \nu_A \nu_B)$, respectively as follows:

$$(\mu_A \circ \mu_B)(x) = \begin{cases} \bigcup_{x=yz} \mu_A(y) \wedge \mu_B(z) & ; \exists y, z \in S, \text{ such that } x = yz \\ 0 & ; \text{ otherwise,} \end{cases}$$

$$(\eta_A \bullet \eta_B)(x) = \begin{cases} \bigcap_{x=yz} \eta_A(y) \vee \eta_B(z) & ; \exists y, z \in S, \text{ such that } x = yz \\ 1 & ; \text{ otherwise,} \end{cases}$$

$$(\nu_A \bullet \nu_B)(x) = \begin{cases} \bigcap_{x=yz} \eta_A(y) \vee \eta_B(z) & ; \exists y, z \in S, \text{ such that } x = yz \\ 1 & ; \text{ otherwise} \end{cases}$$

and

$$(\mu_A \circ \mu_B)(x) = \begin{cases} \bigcup_{x=\sum_{i=1}^n y_i z_i} \left(\bigcap_{i=1}^n \mu_A(y_i) \wedge \bigcap_{i=1}^n \mu_B(z_i) \right) & ; \exists y_i, z_i \in S, \text{ such that } x = \sum_{i=1}^n y_i z_i \\ 0 & ; \text{ otherwise,} \end{cases}$$

$$(\eta_A \eta_B)(x) = \begin{cases} \bigcap_{x=\sum_{i=1}^n y_i z_i} \left(\bigcup_{i=1}^n \eta_A(y_i) \vee \bigcup_{i=1}^n \eta_B(z_i) \right) & ; \exists y_i, z_i \in S, \text{ such that } x = \sum_{i=1}^n y_i z_i \\ 1 & ; \text{ otherwise,} \end{cases}$$

$$(\nu_A \nu_B)(x) = \begin{cases} \bigcap_{x=\sum_{i=1}^n y_i z_i} \left(\bigcup_{i=1}^n \nu_A(y_i) \vee \bigcup_{i=1}^n \nu_B(z_i) \right) & ; \exists y_i, z_i \in S, \text{ such that } x = \sum_{i=1}^n y_i z_i \\ 1 & ; \text{ otherwise.} \end{cases}$$

Theorem 3.1 Let $A = (\mu_A, \eta_A, \nu_A)$, $B = (\mu_B, \eta_B, \nu_B)$ and $C = (\mu_C, \eta_C, \nu_C)$ be any picture fuzzy ideals over a commutative ring R . Then the following properties hold.

1. If $A \subseteq B$, then $C \odot A \subseteq C \odot B$.
2. If $A \subseteq B$, then $CA \subseteq CB$.
3. $C \odot (A \theta B) \subseteq (C \odot A) \theta (C \odot B)$.
4. $C(A \theta B) \subseteq (CA) \theta (CB)$.
5. $C \odot A \subseteq B$ if and only if $CA \subseteq B$.
6. $B \odot A \subseteq A$.
7. If R is a ring with identity, then $R \odot A = A$.

Proof. 1. Let x be an element of R . Then $(\mu_C \circ \mu_A)(x) = \bigcup_{x=ab} \mu_C(a) \wedge \mu_A(b) \leq \bigcup_{x=ab} \mu_C(a) \wedge \mu_B(b) = (\mu_C \circ \mu_B)(x)$ and $(\eta_C \bullet \eta_A)(x) = \bigcap_{x=ab} \eta_C(a) \vee \eta_A(b) \leq \bigcap_{x=ab} \eta_C(a) \vee \eta_B(b) = (\eta_C \bullet \eta_B)(x)$. Similarly, it can be seen that $(\nu_C \bullet \nu_A)(x) \leq (\nu_C \bullet \nu_B)(x)$. Therefore $C \odot A \subseteq C \odot B$.

2. The proof is easy.

3. Let x be an element of R . Then we have

$$\begin{aligned} (\mu_C \circ (\mu_A \oplus \mu_B))(x) &= \bigcup_{x=ab} \mu_C(a) \wedge (\mu_A \oplus \mu_B)(b) \\ &= \bigcup_{x=ab} \mu_C(a) \wedge \left(\bigcup_{b=y+z} \mu_A(y) \wedge \mu_B(z) \right) \\ &= \bigcup_{x=ay+az} (\mu_C(a) \wedge \mu_A(y)) \wedge (\mu_C(a) \wedge \mu_B(z)) \\ &\leq \bigcup_{x=ay+az} (\bigcup_{ay=rs} \mu_C(r) \wedge \mu_A(s)) \wedge (\bigcup_{az=tu} \mu_C(t) \wedge \mu_B(u)) \\ &= \bigcup_{x=c+d} (\mu_C \circ \mu_A)(c) \wedge (\mu_C \circ \mu_B)(d) \\ &= ((\mu_C \circ \mu_A) \oplus (\mu_C \circ \mu_B))(x) \end{aligned}$$

and

$$\begin{aligned} (\eta_C \bullet (\eta_A \otimes \eta_B))(x) &= \bigcap_{x=ab} \eta_C(a) \vee (\eta_A \otimes \eta_B)(b) \\ &= \bigcap_{x=ab} \eta_C(a) \vee (\bigcap_{b=y+z} \eta_A(y) \vee \eta_B(z)) \\ &= \bigcap_{x=ay+az} (\eta_C(a) \vee \eta_A(y)) \vee (\eta_C(a) \vee \eta_B(z)) \\ &\geq \bigcap_{x=ay+az} (\bigcap_{ay=rs} \eta_C(r) \vee \eta_A(s)) \vee (\bigcap_{az=tu} \eta_C(t) \vee \eta_B(u)) \\ &= \bigcap_{x=c+d} (\eta_C \bullet \eta_A)(c) \vee (\eta_C \bullet \eta_B)(d) \\ &= ((\eta_C \bullet \eta_A) \otimes (\eta_C \bullet \eta_B))(x). \end{aligned}$$

Similarly, we obtain that $(\nu_C \bullet (\nu_A \otimes \nu_B))(x) \geq ((\nu_C \bullet \nu_A) \otimes (\nu_C \bullet \nu_B))(x)$. Hence we conclude that $C \odot (A \theta B) \subseteq (C \odot A) \theta (C \odot B)$.

4-5. The proof is easy.

6. Let x be an element of R . Then we have $(\mu_B \circ \mu_A)(x) = \bigcup_{x=ab} \mu_B(a) \wedge \mu_A(b) \leq \bigcup_{x=ab} \mu_A(a) \wedge \mu_A(b) \leq \bigcup_{x=ab} \mu_A(ab) = \mu_A(x)$ and $(\eta_B \bullet \eta_A)(x) = \bigcap_{x=ab} \eta_B(a) \vee \eta_A(b) \geq \bigcap_{x=ab} \eta_A(a) \vee \eta_A(b) \geq \bigcap_{x=ab} \eta_A(ab) = \eta_A(x)$.

Similarly we can show that $(\nu_B \bullet \nu_A)(x) \geq (\nu_A)(x)$. Therefore $B \odot A \subseteq A$.

7. The proof is easy.

Next, we develop some basic properties of the operations \cap and θ .

Theorem 3.2 Let $A = (\mu_A, \eta_A, \nu_A)$ and $B = (\mu_B, \eta_B, \nu_B)$ be any picture fuzzy ideals over a commutative ring R . Then the following properties hold.

1. $A \cap B$ is a picture fuzzy ideal over R .
2. $A \theta B$ is a picture fuzzy ideal over R .

Proof. 1. Let x and y be any elements of R . Then we have

$$\begin{aligned} (\mu_A \wedge \mu_B)(xy) &= \mu_A(xy) \wedge \mu_B(xy) \\ &\geq (\mu_A \wedge \mu_B)(x) \wedge (\mu_A \wedge \mu_B)(y), \\ (\eta_A \vee \eta_B)(xy) &= \eta_A(xy) \vee \eta_B(xy) \\ &\leq (\eta_A \vee \eta_B)(x) \wedge (\eta_A \vee \eta_B)(y), \\ (\mu_A \wedge \mu_B)(x-y) &= \mu_A(x-y) \wedge \mu_B(x-y) \\ &\geq (\mu_A \wedge \mu_B)(x) \wedge (\mu_A \wedge \mu_B)(y) \end{aligned}$$

$$\begin{aligned} \text{and } (\eta_A \vee \eta_B)(x-y) &= \eta_A(x-y) \vee \eta_B(x-y) \\ &\leq (\eta_A \vee \eta_B)(x) \vee (\eta_A \vee \eta_B)(y). \end{aligned}$$

Similarly, it can be similarly proved that $(\nu_A \vee \nu_B)(xy) \leq (\nu_A \vee \nu_B)(x) \wedge (\nu_A \vee \nu_B)(y)$ and $(\nu_A \vee \nu_B)(x-y) \leq (\nu_A \vee \nu_B)(x) \vee (\nu_A \vee \nu_B)(y)$. Thus we have $A \cap B$ is a picture fuzzy ideal over R .

2. Let x and y be any elements of R . Then we have

$$\begin{aligned}
 (\mu_A \oplus \mu_B)(x - y) &= \bigcup_{x-y=a+b} \mu_A(a) \wedge \mu_B(b) \\
 &\geq \bigcup_{x=a_1+a_2, y=b_1+b_2} \mu_A(a_1 - b_1) \wedge \mu_B(a_2 - b_2) \\
 &\geq \left(\bigcup_{x=a_1+a_2} \mu_A(a_1) \wedge \mu_B(a_2) \right) \wedge \left(\bigcup_{y=b_1+b_2} \mu_A(b_1) \wedge \mu_B(b_2) \right) \\
 &= (\mu_A \oplus \mu_B)(x) \wedge (\mu_A \oplus \mu_B)(y) \\
 (\eta_A \otimes \eta_B)(x - y) &= \bigcap_{x-y=a+b} \eta_A(a) \vee \eta_B(b) \\
 &\leq \bigcap_{x=a_1+a_2, y=b_1+b_2} \eta_A(a_1 - b_1) \vee \eta_B(a_2 - b_2) \\
 &\leq \left(\bigcap_{x=a_1+a_2} \eta_A(a_1) \vee \eta_B(a_2) \right) \vee \left(\bigcap_{y=b_1+b_2} \eta_A(b_1) \vee \eta_B(b_2) \right) \\
 &= (\eta_A \otimes \eta_B)(x) \vee (\eta_A \otimes \eta_B)(y) \\
 (\mu_A \oplus \mu_B)(xy) &= \bigcup_{xy=ab} \mu_A(a) \wedge \mu_B(b) \\
 &\geq \bigcup_{y=a_1+a_2} (\mu_A(x) \wedge \mu_B(y)) \vee (\mu_A(a_1) \wedge \mu_B(a_2)) \\
 &= (\mu_A \oplus \mu_B)(x) \vee (\mu_A \oplus \mu_B)(y) \\
 (\eta_A \otimes \eta_B)(xy) &= \bigcap_{xy=a+b} \eta_A(a) \vee \eta_B(b) \\
 &\leq \bigcap_{y=a_1+a_2} (\eta_A(x) \vee \eta_B(y)) \wedge (\eta_A(a_1) \vee \eta_B(a_2)) \\
 &= (\eta_A \otimes \eta_B)(x) \wedge (\eta_A \otimes \eta_B)(y).
 \end{aligned}$$

Similarly we can see that $(\nu_A \otimes \nu_B)(x - y) \leq (\nu_A \otimes \nu_B)(x) \vee (\nu_A \otimes \nu_B)(y)$ and $(\nu_A \otimes \nu_B)(xy) \leq (\nu_A \otimes \nu_B)(x) \wedge (\nu_A \otimes \nu_B)(y)$. Thus we have, $A \theta B$ is a picture fuzzy ideal over R .

4. CONCLUSION

In the structural theory of picture fuzzy algebraic systems, picture fuzzy sets with special properties always play an important role. In this work, we focus on a particular topic related to picture fuzzy algebra, which develops picture fuzzy versions of commutative rings. Specifically, we study the theory of fuzzy sets and picture fuzzy sets. We introduce the notions of picture fuzzy ideals on commutative rings and some properties of them are obtained. Finally, we give suitable definitions of the operations of picture fuzzy ideals over a commutative ring, as composition, product and intersection.

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