Hardware impairments aware full-duplex non-orthogonal multiple access networks over Nakagami-m channels

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Article Info

Article history:

Received Apr 4, 2021
Revised Dec 10, 2021
Accepted Feb 22, 2022

Keywords:

Hardware impairments
Non-orthogonal multiple access
Outage probability

ABSTRACT

Non-orthogonal multiple access (NOMA) and full-duplex (FD) relaying communications are promising candidates for 5G cellular networks. In this paper, by exploiting the impact of hardware impairment, we study FD NOMA communications with a downlink scheme. In a group of two users, we find that the target rates and power allocation strategies are main factors affecting the system performance metric. We derive the closed-form formula of outage probability for two users. As main contribution, numerical results are considered to illustrate the performance of the FD NOMA. We also study the base station (BS) can adjust its transmit signal to noise ratio (SNR) to achieve relevant outage probability in several scenarios.

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1. INTRODUCTION

As one of promising approach implemented in 5G systems, non-orthogonal multiple access (NOMA) was introduced [1]. To provide massive connections, NOMA can rely on this first main benefit to further serve services with high spectral efficiency and low latency. Those advances are urgent requirements to design new generation of 5G and beyond wireless systems [2]. In NOMA, the multiple users can be shared same frequency but different power levels are assigned to each user effectively. The signal detection technique is required at receiver to extract information exactly with low error. How NOMA treats far users and near users to assign power levels. Fortunately, by detecting the channel gains of different channels, suitable power coefficients are assigned to users reasonably [3]–[5]. As interesting application of NOMA techniques, half-duplex relay stations (RSs) have been studied in order to increase spatial diversity [6]–[13]. The benefit of relay can be reported in [8], [10], and [11], since single relay is placed between transmitters and receivers. Nakagami-m fading channels [8] and Rayleigh fading channels [11] are popular channel models deployed in the NOMA system relying on a single amplify-and-forward (AF) relay, which outperform the orthogonal multiple access (OMA) in terms of two system performance metrics (outage probability and throughput). Kader *et al.* in [10] developed a network containing two sources, two destinations, and a relay to form the cooperative NOMA with a half-duplex decode-and-forward (DF). They examined perfect and imperfect successive interference cancellation (SIC) when they evaluated ergodic sum capacity.

As simpler approach, Liang *et al.* and Xu *et al.* in [14], [15] studied half-duplex (HD) relay-based NOMA systems. Due to the requirement of additional time resources, HD NOMA just provides low spectral efficiency. Different from HD NOMA, full-duplex (FD) relay-based NOMA deploys the same frequency channel to permit the relay to simultaneously receive and transmit signals and reduce such loss [16]–[19]. The operation of FD benefits from advances of antenna isolation and cancellation of analog self-interference (SI) at the FD relay. FD device-to-device assisted coop erative NOMA system was investigated [20] in which the near user needs the FD relay to support transmission to the far user. Deng *et al.* [21] adopted Rician fading channels for FD NOMA system by evaluating formulas of the outage probability and the ergodic rate under imperfect conditions such as imperfect SIC and residual hardware impairments at transceivers [22] considered the impact of imperfect SIC and residual inter-relay interference on a DF relaying based NOMA. The authors developed for the considered framework over generalized Nakagami-m fading channels by evaluating outage probability (OP), asymptotic OP, and ergodic rate. The work in [23] studied downlink NOMA short-packet communication systems the average block error rate (BLER) by using stochastic geometry and Nakagami-m fading channels. A few work consider FD at relay for NOMA, for example [24], which motives us to study difference among two destinations under the impact of hardware impairments.

2. SYSTEM MODEL

A dual-hop NOMA transmission with the help of a FD relay (R) is studied, shown in Figure. 1. The system model could be examined in the case of a base station (B) serves a dedicated group of two NOMA users. In particular, we design a FD relay (R) which is intermediate device while two NOMA users including D_1 and D_2. Those users are classified based on channel gains to determine the near and the far users. The FD relay is equipped two antennas to transmit and receive signals simultaneously. To provide general channel model, all the channels are assumed as Nakagami-m channels. We treat power coefficients $\varepsilon_1, \varepsilon_2$ to help the base station B serving dedicated group of users and satisfying strict constraints $\varepsilon_1 + \varepsilon_2 = 1$ and $\varepsilon_1 > \varepsilon_2$.

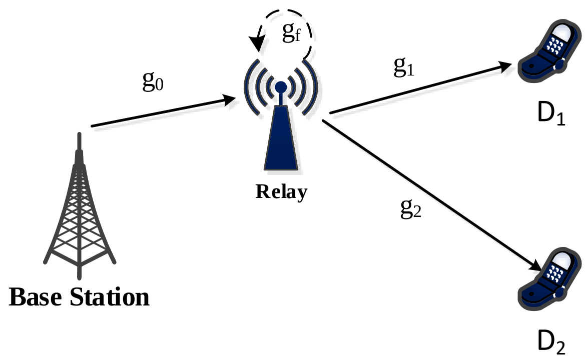

Figure 1. Considering hardware impairment aware FD NOMA system

The transmit signal processed at the base station B is $\sqrt{\varepsilon_1 \mathrm{P_B}} x_1 + \sqrt{\varepsilon_2 \mathrm{P_B}} x_2$, in which $\mathrm{P_B}$ is the total transmitted power of B; x_1, x_2 are denoted as signals of $\mathrm{D}_1, \mathrm{D}_2$.

The transmit signal from the base station is then processed at the FD relay. It is worth noting that FD relay produces x_{g_f} as the loop self-interference which is expected to eliminate. In particular, we can compute the received signal at the relay as:

$$y_{\mathrm{R}} = g_0 \left(y_{\mathrm{B}} + \eta_{\mathrm{S}} \right) + g_f \xi \left(\sqrt{\mathrm{P_R}} x_{g_f} + \eta_{\mathrm{R}} \right) + \eta_0, \quad (1)$$

where g_0, g_f are the channel coefficients of $\mathrm{B} \to \mathrm{R}$ and $\mathrm{R} \to \mathrm{R}$ links. ξ is denoted for HD/FD modes, i.e. $\xi = 0$ and $\xi = 1$ are known as HD and FD modes respectively. $\eta_{\mathrm{B}} \sim \Gamma \left(0, \kappa_{\mathrm{BR}}^2 \mathrm{P_B} |g_0|^2 \right)$, $\eta_{\mathrm{R}} \sim \Gamma \left(0, \kappa_{g_f}^2 \mathrm{P_R} |g_f|^2 \right)$, $\eta_{\mathrm{D}_i} \sim \Gamma \left(0, \kappa_{\mathrm{D}_i}^2 \mathrm{P_R} |g_i|^2 \right), (i = 1, 2)$ represents noise distortion, $\eta_0 \sim \Gamma \left(0, \mathrm{N_0} \right)$ denotes the additive white Gaussian noise. κ_{BR}, κ_{g_f} and $\kappa_{\mathrm{D}_i}, (i = 1, 2)$ are the levels of residual hardware impairments.

The second hop signal processing is conducted based on channels g_i which is refereed to links $\mathrm{R} \rightarrow \mathrm{D}_i$. The NOMA power allocation need be adjusted for the second hop transmission, i.e. μ_1, μ_2 are allocated to two signals of two users, those factors are satisfied $\mu_1 + \mu_2 = 1$ and $\mu_1 > \mu_2$.

The received signal at $\mathrm{D}_1, \mathrm{D}_2$ are given as:

$$y_{\mathrm{D}_i} = g_i \left(\sqrt{\mu_1 \mathrm{P_R}} x_1 + \sqrt{\mu_2 \mathrm{P_R}} x_2 + \eta_{\mathrm{D}_i} \right) + \eta_0, \tag{2}$$

The two signals x_1, x_2 need be processed at the FD relay based on signal to interference plus noise ratio (SINR) which can be computed by:

$$\gamma_{x_1,\mathrm{R}} = \frac{\varepsilon_1 \rho_{\mathrm{B}} |g_0|^2}{(\varepsilon_2 + \kappa_{\mathrm{BR}}^2)\, \rho_{\mathrm{B}} |g_0|^2 + (1 + \kappa_{\mathrm{BR}}^2)\, \rho_{\mathrm{R}} \xi^2 |g_f|^2 + 1}, \tag{3}$$

and

$$\gamma_{x_2,\mathrm{R}} = \frac{\varepsilon_2 \rho_{\mathrm{B}} |g_0|^2}{\rho_{\mathrm{B}} \kappa_{\mathrm{BR}}^2 |g_0|^2 + (1 + \kappa_{\mathrm{BR}}^2)\, \rho_{\mathrm{R}} \xi^2 |g_f|^2 + \varepsilon_1 \rho_{\mathrm{B}} h_{\mathrm{R}} + 1}, \tag{4}$$

where $\rho_{\mathrm{B}} = \frac{\mathrm{P_B}}{N_0}$, $\rho_{\mathrm{R}} = \frac{\mathrm{P_R}}{N_0}$ are the transmit SNR at B and R. $h_{\mathrm{R}} \sim \Gamma\left(0, \omega |g_0|^2\right)$ caused by imperfect SIC (ipSIC) and $\omega \in [0, 1)$.

After signals transmitted at the second hop transmission, the destination D_2 need to know SINR as below. In particular, D_2 detects signal x_1 as shown in:

$$\gamma_{x_1,\mathrm{D}_2} = \frac{\mu_1 \rho_{\mathrm{R}} |g_2|^2}{\mu_2 \rho_{\mathrm{R}} |g_2|^2 + \rho_{\mathrm{R}} \kappa_{\mathrm{D}_2}^2 |g_2|^2 + 1}. \tag{5}$$

The first user D_1 wants to detect signal x_1, x_2 respectively as shown in:

$$\gamma_{x_1,\mathrm{D}_1} = \frac{\mu_1 \rho_{\mathrm{R}} |g_1|^2}{\mu_2 \rho_{\mathrm{R}} |g_1|^2 + \rho_{\mathrm{R}} \kappa_{\mathrm{D}_1}^2 |g_1|^2 + 1}, \tag{6}$$

and

$$\gamma_{x_2,\mathrm{D}_1} = \frac{\mu_2 \rho_{\mathrm{R}} |g_1|^2}{\mu_1 \rho_{\mathrm{R}} h_{\mathrm{RD}_1} + \rho_{\mathrm{R}} \kappa_{\mathrm{D}_1}^2 |g_1|^2 + 1}, \tag{7}$$

where $h_{\mathrm{RD}_1} \sim \Gamma\left(0, \omega |g_1|^2\right)$.

The PDF of the Nakagami-m channel gain g_k $(k = 0, 1, 2, f)$ can be expressed as:

$$f_{|g_k|^2}(x) = \frac{x^{m_{g_k} - 1}}{\Gamma(m_{g_k})\, \beta_{g_k}^{m_{g_k}}} \mathrm{e}^{-\frac{x}{\beta_{g_k}}}, \tag{8}$$

where $\beta_{g_k} = {}^{\lambda_{g_k}}\!/_{m_{g_k}}$ is the mean value of g_k denoted by $|g_k|^2 \sim \Gamma\left(m_{g_k}, \frac{\lambda_{g_k}}{m_{g_k}}\right)$. The CDF can be written by:

$$F_{|g_k|^2}(x) = 1 - \frac{1}{\Gamma(m_{g_k})} \Gamma\left(m_{g_i}, \frac{x}{\beta_{g_k}}\right) = 1 - \mathrm{e}^{-\frac{x}{\beta_{g_k}}} \sum_{n=0}^{m_{g_k}-1} \frac{x^n}{n! \beta_{g_k}^n}. \tag{9}$$

3. PERFORMANCE ANALYSIS

3.1. Outage probability of D_1

To evaluate system performance, the OP need be compute, the OP of D_1 is defined as [19], [20].

$$\begin{aligned} \mathrm{OP}_{\mathrm{D}_1} = & \Pr\left(\min\left(\gamma_{x_1,\mathrm{R}} < \gamma_2^{\mathrm{th}}, \gamma_{x_2,\mathrm{R}} < \gamma_1^{\mathrm{th}}\right)\right) + \Pr\left(\begin{array}{c} \min\left(\gamma_{x_1,\mathrm{R}} \geq \gamma_2^{\mathrm{th}}, \gamma_{x_2,\mathrm{R}} \geq \gamma_1^{\mathrm{th}}\right), \\ \min\left(\gamma_{x_1,\mathrm{D}_1} < \gamma_2^{\mathrm{th}}, \gamma_{x_2,\mathrm{D}_1} < \gamma_1^{\mathrm{th}}\right) \end{array} \right) \\ = & \theta_1 + \theta_2 \end{aligned} \tag{10}$$

where the threshold SNRs are $\gamma_1^{\rm th} = 2^{R_1} - 1, \gamma_2^{\rm th} = 2^{R_2} - 1$.

Replacing the formulas (3), (4) into (10), we can calculate θ_1 as shown in:

$$\begin{aligned} \theta_1 \triangleq & \Pr\left(|g_0|^2 < \frac{\Phi\gamma_2^{\rm th}|g_f|^2}{\varphi_1} + \frac{\gamma_2^{\rm th}}{\varphi_1}, |g_f|^2 < \Delta_1\right) + \Pr\left(|g_0|^2 < \frac{\Phi\gamma_1^{\rm th}|g_f|^2}{\varphi_2} + \frac{\gamma_1^{\rm th}}{\varphi_2}, |g_f|^2 \geq \Delta_1\right) \\ & = \tau_1 + \tau_2, \end{aligned} \tag{11}$$

where $\varphi_1 \triangleq \varepsilon_1\rho_{\rm B} - \left(\varepsilon_2 + \kappa_{\rm BR}^2\right)\rho_{\rm B}\gamma_2^{\rm th}$, $\varphi_2 \triangleq \varepsilon_2\rho_{\rm B} - \left(\varepsilon_1\omega + \kappa_{\rm BR}^2\right)\rho_{\rm B}\gamma_1^{\rm th}$, $\Phi \triangleq \left(1 + \kappa_{\rm BR}^2\right)\rho_{\rm R}\xi^2$, $\Delta_1 \triangleq \frac{\gamma_1^{\rm th}\varphi_1 - \gamma_2^{\rm th}\varphi_2}{\Phi\gamma_2^{\rm th}\varphi_2 - \Phi\gamma_1^{\rm th}\varphi_1}$ and we can be calculated τ_1, τ_2 as:

$$\begin{aligned} \tau_1 \triangleq & \Pr\left(|g_0|^2 < \frac{\Phi\gamma_2^{\rm th}|g_f|^2}{\varphi_1} + \frac{\gamma_2^{\rm th}}{\varphi_1}, |g_f|^2 < \Delta_1\right) \\ & = \int_0^{\Delta_1} \frac{x^{m_{g_f}-1}}{\Gamma\left(m_{g_f}\right)\beta_{g_f}^{m_{g_f}}} {\rm e}^{-\frac{x}{\beta_{g_f}}} dx - \sum_{n=0}^{m_{g_0}-1} \int_0^{\Delta_1} \left(\frac{\Phi\gamma_2^{\rm th}x}{\varphi_1} + \frac{\gamma_2^{\rm th}}{\varphi_1}\right)^n \alpha_1 x^{m_{g_f}-1} {\rm e}^{-\alpha_2 x} dx \\ & = \frac{1}{\Gamma\left(m_{g_f}\right)}\gamma\left(m_{g_f}, \frac{\Delta_1}{\beta_{g_f}}\right) - \sum_{n=0}^{m_{g_0}-1}\sum_{k=0}^{n} \binom{n}{k} \left(\frac{\gamma_2^{\rm th}}{\varphi_1}\right)^n \alpha_1 \alpha_2^{-k-m_{g_f}} \Phi^k \gamma\left(\left(k + m_{g_f}\right), \alpha_2\Delta_1\right). \end{aligned} \tag{12}$$

In which, $\alpha_1 \triangleq \frac{{\rm e}^{-\frac{\gamma_2^{\rm th}}{\varphi_1\beta_{g_0}}}}{n!\beta_{g_0}^n\Gamma\left(m_{g_f}\right)\beta_{g_f}^{m_{g_f}}}$, $\alpha_2 \triangleq \frac{\Phi\gamma_2^{\rm th}}{\varphi_1\beta_{g_0}} + \frac{1}{\beta_{g_f}}$, $\alpha_5 \triangleq \frac{{\rm e}^{-\frac{\gamma_1^{\rm th}}{\varphi_2\beta_{g_0}}}}{n!\beta_{g_0}^n\Gamma\left(m_{g_f}\right)\beta_{g_f}^{m_{g_f}}}$, $\alpha_6 \triangleq \frac{\Phi\gamma_1^{\rm th}}{\varphi_2\beta_{g_0}} + \frac{1}{\beta_{g_f}}$ and τ_2 is computed by.

$$\begin{aligned} \tau_2 \triangleq & \Pr\left(|g_0|^2 < \frac{\Phi\gamma_1^{\rm th}|g_f|^2}{\varphi_2} + \frac{\gamma_1^{\rm th}}{\varphi_2}, |g_f|^2 \geq \Delta_1\right) \\ & = \int_{\Delta_1}^{\infty} \frac{x^{m_{g_f}-1}}{\Gamma\left(m_{g_f}\right)\beta_{g_f}^{m_{g_f}}} {\rm e}^{-\frac{x}{\beta_{g_f}}} dx - \int_{\Delta_1}^{\infty} \sum_{n=0}^{m_{g_0}-1}\sum_{k=0}^{n} \binom{n}{k} \left(\frac{\gamma_1^{\rm th}}{\varphi_2}\right)^n \alpha_5 \Phi^k x^{k+m_{g_f}-1} {\rm e}^{-\alpha_6 x} dx \\ & = \beta_{g_f}^{m_{g_f}} \Gamma\left(m_{g_f}, \frac{\Delta_1}{\beta_{g_f}}\right) - \sum_{n=0}^{m_{g_0}-1}\sum_{k=0}^{n} \binom{n}{k} \left(\frac{\gamma_1^{\rm th}}{\varphi_2}\right)^n \alpha_5 \alpha_6^{-k-m_{g_f}} \Phi^k \Gamma\left(\left(k + m_{g_f}\right), \alpha_6\Delta_1\right). \end{aligned} \tag{13}$$

Similarly θ_1, θ_2 can be calculated as shown in:

$$\begin{aligned} \theta_2 \triangleq & \Pr\left(\min\left(\gamma_{x_1,{\rm R}} \geq \gamma_2^{\rm th}, \gamma_{x_2,{\rm R}} \geq \gamma_1^{\rm th}\right)\right) \Pr\left(\min\left(\gamma_{x_1,{\rm D_1}} < \gamma_2^{\rm th}, \gamma_{x_2,{\rm D_1}} < \gamma_1^{\rm th}\right)\right) \\ & = \tau_3 \times \tau_4. \end{aligned} \tag{14}$$

We can calculate τ_3 and τ_4 as shown in:

$$\begin{aligned} \tau_3 \triangleq & 1 - \Pr\left(|g_0|^2 < \min\left(\frac{\Delta_4\gamma_2^{\rm th}}{\Delta_2}|g_f|^2 + \frac{\gamma_2^{\rm th}}{\Delta_2}, \frac{\Delta_4\gamma_1^{\rm th}}{\Delta_3}|g_f|^2 + \frac{\gamma_1^{\rm th}}{\Delta_3}\right)\right) \\ & = 1 - \Psi_1 - \Psi_2 \end{aligned} \tag{15}$$

where $\Delta_2 \triangleq \varepsilon_1\rho_{\rm B} - \left(\varepsilon_2 + \kappa_{\rm BR}^2\right)\rho_{\rm B}\gamma_2^{\rm th}$, $\Delta_3 \triangleq \varepsilon_2\rho_{\rm B} - \rho_{\rm B}\kappa_{\rm BR}^2\gamma_1^{\rm th} - \varepsilon_1\rho_{\rm B}\omega\gamma_1^{\rm th}$, $\Delta_4 \triangleq \left(1 + \kappa_{\rm BR}^2\right)\rho_{\rm R}\xi^2$,

$\alpha_3 \triangleq \left(\frac{\gamma_2^{\rm th}}{\Delta_2}\right)^n \frac{{\rm e}^{-\frac{\gamma_2^{\rm th}}{\Delta_2\beta_{g_0}}}\Delta_4^{k_1}}{n!\beta_{g_0}^n\Gamma(m_{g_f})\beta_{g_f}^{m_{g_f}}}$, $\Theta \triangleq \frac{\Delta_2\gamma_1^{\rm th}-\Delta_3\gamma_2^{\rm th}}{\Delta_3\Delta_4\gamma_2^{\rm th}-\Delta_2\Delta_4\gamma_1^{\rm th}}$, $\alpha_4 \triangleq \frac{\Delta_4\gamma_2^{\rm th}}{\Delta_2\beta_{g_0}}+\frac{1}{\beta_{g_f}}$,

$$\begin{aligned}\Psi_1 &\triangleq {\rm Pr}\left(|g_0|^2<\frac{\Delta_4\gamma_2^{\rm th}}{\Delta_2}|g_f|^2+\frac{\gamma_2^{\rm th}}{\Delta_2},|g_f|^2<\Theta\right)\\ &=\int_0^\Theta \frac{x^{m_{g_f}-1}}{\Gamma(m_{g_f})\beta_{g_f}^{m_{g_f}}}{\rm e}^{-\frac{x}{\beta_{g_f}}}dx-\int_0^\Theta\sum_{n=0}^{m_{g_0}-1}\sum_{k_1}^{n}\binom{n}{k_1}\alpha_3x^{k_1+m_{g_f}-1}{\rm e}^{-\alpha_4x}dx\\ &=\frac{\gamma\left(m_{g_f},\frac{\Theta}{\beta_{g_f}}\right)}{\Gamma(m_{g_f})}-\sum_{n=0}^{m_{g_0}-1}\sum_{k_1}^{n}\binom{n}{k_1}\alpha_4^{-k_1-m_{g_f}}\alpha_3\gamma\left(\left(k_1+m_{g_f}\right),\alpha_4\Theta\right),\end{aligned} \tag{16}$$

$$\begin{aligned}\Psi_2 &\triangleq {\rm Pr}\left(|g_0|^2<\frac{\Delta_4\gamma_1^{\rm th}}{\Delta_3}|g_ff|^2+\frac{\gamma_1^{\rm th}}{\Delta_3},|g_f|^2\ge\Theta\right)\\ &=\frac{\Gamma\left(m_{g_f},\frac{\Theta}{\beta_{g_f}}\right)}{\Gamma(m_{g_f})}-\sum_{n=0}^{m_{g_0}-1}\sum_{k_1}^{n}\binom{n}{k_1}\left(\frac{\gamma_2^{\rm th}}{\Delta_3}\right)^n\frac{\Delta_4^{k_1}{\rm e}^{-\frac{\gamma_2^{\rm th}}{\Delta_3\beta_{g_0}}}}{n!\beta_{g_0}^n\Gamma(m_{g_f})\beta_{g_f}^{m_{g_f}}}\\ &\times\Gamma\left(k_1+m_{g_f},\frac{\Delta_4\gamma_2^{\rm th}\Theta}{\Delta_3\beta_{g_0}}+\frac{\Theta}{\beta_{g_f}}\right)\left(\frac{\Delta_4\gamma_2^{\rm th}}{\Delta_3\beta_{g_0}}+\frac{1}{\beta_{g_f}}\right)^{-k_1-m_{g_f}}.\end{aligned} \tag{17}$$

Then, τ_4 can be expressed as:

$$\begin{aligned}\tau_4 &\triangleq {\rm Pr}\left(\min\left(\frac{\mu_1\rho_{\rm R}|g_1|^2}{\mu_2\rho_{\rm R}|g_1|^2+\rho_{\rm R}\kappa_{{\rm D}_1}^2|g_1|^2+1}<\gamma_2^{\rm th},\frac{\mu_2\rho_{\rm R}|g_1|^2}{\mu_1\rho_{\rm R}\omega|g_1|^2+\rho_{\rm R}\kappa_{{\rm D}_1}^2|g_1|^2+1}<\gamma_1^{\rm th}\right)\right)\\ &={\rm Pr}\left(|g_1|^2<\min(\Delta_5,\Delta_6)\right)\\ &=1-{\rm e}^{-\frac{\min(\Delta_5,\Delta_6)}{\beta_{g_1}}}\sum_{n=0}^{m_{g_1}-1}\frac{(\min(\Delta_5,\Delta_6))^n}{n!\beta_{g_1}^n}.\end{aligned} \tag{18}$$

with $\Delta_5 \triangleq \frac{\gamma_2^{\rm th}}{\mu_1\rho_{\rm R}-\mu_2\rho_{\rm R}\gamma_2^{\rm th}-\rho_{\rm R}\kappa_{{\rm D}_1}^2\gamma_2^{\rm th}}$, $\Delta_6 \triangleq \frac{\gamma_1^{\rm th}}{\mu_2\rho_{\rm R}-\mu_1\rho_{\rm R}\omega\gamma_1^{\rm th}-\rho_{\rm R}\kappa_{{\rm D}_1}^2\gamma_1^{\rm th}}$.

We have applied the formulas [25], (1.111), [25], (3.381.1), and [25], (3.381.3) in the calculation steps above.

3.2. Outage probability of D_2

The OP of D_2 can be written as:

$$\begin{aligned}{\rm OP}_{{\rm D}_2} &={\rm Pr}\left(\gamma_{x_1,{\rm R}}<\gamma_2^{\rm th},\gamma_{x_1,{\rm D}_2}<\gamma_2^{\rm th},\gamma_{x_2,{\rm R}}<\gamma_1^{\rm th}\right)\\ &={\rm Pr}\left(\gamma_{x_1,{\rm R}}<\gamma_2^{\rm th},\gamma_{x_2,{\rm R}}<\gamma_1^{\rm th}\right)\times{\rm Pr}\left(\gamma_{x_1,{\rm D}_2}<\gamma_2^{\rm th}\right)\\ &=\theta_1\times\theta_3,\end{aligned} \tag{19}$$

where θ_1 was calculated in the previous section and afer substituting (3) into (19), θ_3, we obtain:

$$\begin{aligned}\theta_3 &\triangleq {\rm Pr}\left(|g_2|^2<\frac{\gamma_2^{\rm th}}{\mu_1\rho_{\rm R}-\mu_2\rho_{\rm R}\gamma_2^{\rm th}\rho_{\rm R}\kappa_{{\rm D}_2}^2\gamma_2^{\rm th}}\right)\\ &=1-{\rm e}^{-\frac{\gamma_2^{\rm th}}{\mu_1\rho_{\rm R}\beta_{g_1}-\mu_2\rho_{\rm R}\beta_{g_1}\gamma_2^{\rm th}-\rho_{\rm R}\kappa_{{\rm D}_2}^2\beta_{g_1}\gamma_2^{\rm th}}}\sum_{n=0}^{m_{g_1}-1}\frac{\left(\frac{\gamma_2^{\rm th}}{\mu_1\rho_{\rm R}-\mu_2\rho_{\rm R}\gamma_2^{\rm th}-\rho_{\rm R}\kappa_{{\rm D}_2}^2\gamma_2^{\rm th}}\right)^n}{n!\beta_{g_1}^n}.\end{aligned} \tag{20}$$

4. SIMULATION RESULTS

In this section, we assume that the levels of RHIs $\kappa = \kappa_{\mathrm{BR}} = \kappa_{g_f} = \kappa_{\mathrm{D}_1} = \kappa_{\mathrm{D}_2}$, the mean values of channel power gains $\lambda_{g_0} = \lambda_{g_2}$, λ_{g_1}, λ_{g_f}, the target rates of D_1, D_2 are respectively R_1, R_2, $\omega = 0.01$ and power allocation coefficients $\varepsilon_1 = \mu_1$, $\varepsilon_2 = \mu_2$. The better quality of channels (higher m) leads to improvement of OP performance for two users, shown in Figure 2. In addition, in Figure 3, higher requirement of data rate R_1, R_2 results in worse OP performance. The reason is that in (10), OP depends on the target rates.

We then see the impact of level of self-interference channel at the relay on OP in Figure 4 performance. $\lambda_{g_f} = 0.1$ is reported as the best case for two users. The difference among two users is decided by different power allocation factor assigned. The impact of hardware impairment can be observed in Figure 5. Less impact of hardware impairment $\kappa = 0.001$ is the best OP performance.

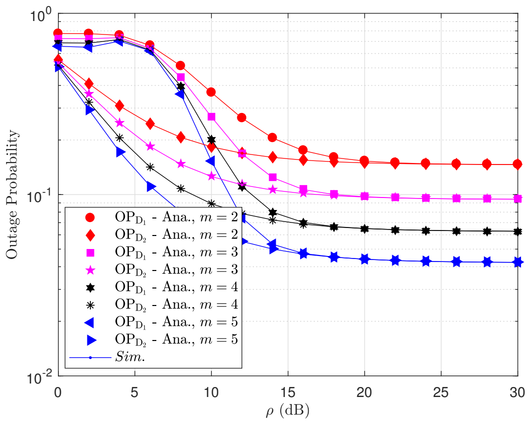

Figure 2. Outage probability versus transmit SNR (ρ) with different m with R_1 = 2 (bits/s/Hz), R_2 = 1 (bits/s/Hz), $\varepsilon_1 = \mu_1 = 0.6$, $\kappa = 0.05$, $\lambda_{g_0} = \lambda_{g_2} = 8$, $\lambda_{g_1} = 1$, $\lambda_{g_f} = 0.5$

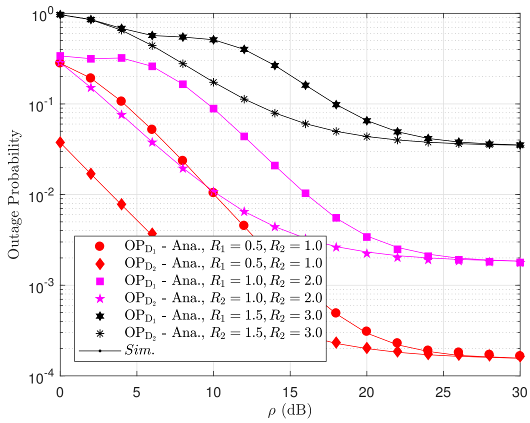

Figure 3. Outage probability versus transmit signal to noise ratio (SNR) (ρ) with different target rates, $\varepsilon_1 = \mu_1 = 0.7$, $\kappa = 0.05$, $\lambda_{g_0} = \lambda_{g_2} = 5$, $\lambda_{g_1} = 1$, $\lambda_{g_f} = 0.5$,m = 2

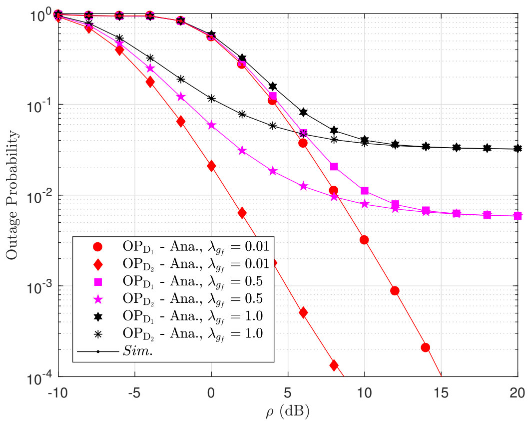

Figure 4. Outage probability versus transmit SNR (ρ) with different of λ_{g_f}, R_1 = 0.5 (bits/s/Hz), R_2 = 0.5 (bits/s/Hz), $\lambda_{g_0} = \lambda_{g_2} = 5$, $\lambda_{g_1} = 1$, m = 3, $\varepsilon_1 = \mu_1 = 0.6$, $\kappa = 0.01$

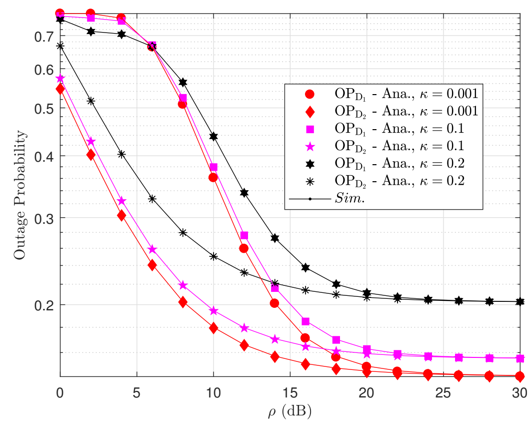

Figure 5. Outage probability versus transmit SNR (ρ) with different of κ, R_1 = 2 (bits/s/Hz), R_2 = 1 (bits/s/Hz), $\varepsilon_1 = \mu_1 = 0.66$, $\lambda_{g_0} = \lambda_{g_2} = 8$, $\lambda_{g_1} = 1$, $\lambda_{g_f} = 0.5$,m = 2

5. CONCLUSION

In this article, a downlink FD NOMA system was studied under the impact of hardware impairment. To illustrate advantage of NOMA scheme, the closed-form expressions of outage probability were provided. Numerical results were presented to corroborate the theoretical analysis, demonstrating that the quality of channel, level of hardware noise yield significant performance gains over Nakagami-m fading. Moreover, all the results showed that the system performance is limited by the target rates. NOMA with more users can be addressed in the future work.

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