

Illustrate the Butterfly Effect on the Chaos Rikitake system

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Abstract

This letter presents butterfly effect on a Chaos system. In this letter we want to briefly introduce Chaos Rikitake system and monitor the butterfly effect on this system. In chaos theory, the butterfly effect is the sensitive dependency on initial conditions. For this goal at the first we suppose initiation point and plot it, for base of work, later will apply small change on one item of initiation point and monitor behavior of Rikitake system. At the end we want to reclaim the famous lecture of Edward Lorenz in 1972 "Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?". The numerical simulations by use of MATLAB software are given to illustrate the butterfly effect on this system.

Keywords: Chaos; Rikitake system; butterfly effect; initiation point

1. Introduction

This system is a mathematical model obtained from a simple mechanical system used by Rikitake [1] to study the reversals of the Earth's magnetic field. This model attempts to explain the reversal of the Earth's magnetic [2]. This system describes the currents of two coupled dynamo disks. The governing equations are:

$$\begin{cases} \dot{x}_1 = -ux + x_2x_3 \\ \dot{x}_2 = -ux_2 + (x_3 - a)x_1 \\ \dot{x}_3 = 1 - x_1x_2 \end{cases} \quad (1)$$

The choice of the parameters $a > 0$ and $u > 0$ reflects a physical meaning in the Rikitake model. For study physical meaning can see [3]. We obtain a & u by use of bellow formulas:

$$a = R\sqrt{\frac{LC}{GM}} \quad \& \quad u = (\omega_1 - \omega_2)\sqrt{\frac{CM}{GL}} \quad (2)$$

where L , R are the self-inductance and resistance of the coil, the angular velocity is ω , and momentum of inertia is C , and the driving force is G , M and N are the mutual inductance between the coils and the disks.

2. Numerical Simulations

In this section we want to plot the behavior of Rikitake system by use of values that we will suppose.

For a & u :

These two parameters are strongly dependent on the physical structure of the system. Because we have a chaotic system see [4] we suppose $a = 3$ and $u = 1.3$.

At the first we plot a model base, after that will change initiation point at three steps and monitor the changing.

The parameters have been set to $a = 3$ and $u = 1.3$. The initiation conditions $(x_1, x_2, x_3) = (1.1453130, 1.1453130, 1.1453130)$ have been used. The numerical solution has been approximated from $t = 0$ to $t = 300$ and the numerical solutions have been saved and plotted at intervals of 0.01, i.e. at times $t = 0, 0.01, 0.02, \dots, 300$. In all numerical runs, the solution has been approximated at $\Delta t = 0.01$.

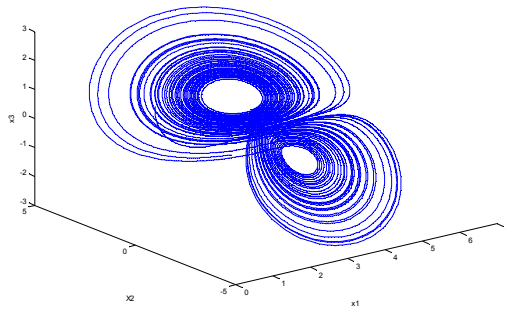


Figure 1. Behavior of Rikitake system in 3D plot

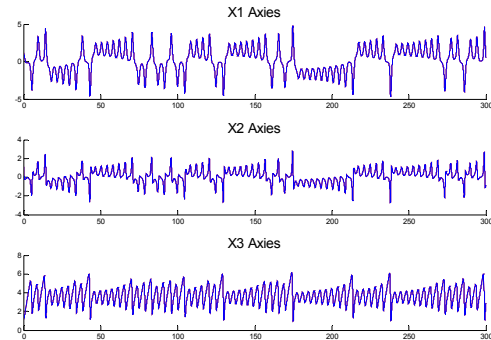


Figure 2. Behavior of Rikitake system for base model

We have a chaotic system, as the base model to continue working. At the next in three steps will investigate Butterfly effect on Rikitake system.

Step 1

The parameters have been set to $a = 3$ $u = 1.3$ and The numerical solution has been approximated from $t = 0$ to $t = 300$. Plot the behavior of system for $(x_1, x_2, x_3) = (1.14531301, 1.14531301, 1.14531301)$.

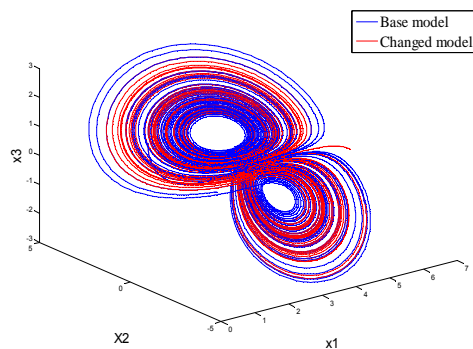


Figure 3. Difference behavior of Rikitake system in 3D plot for first mode

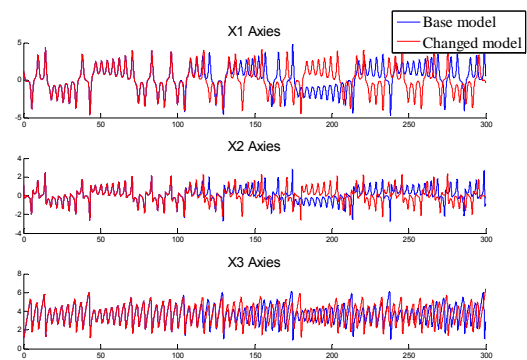


Figure 4. Difference between base and changed model for first mode

We plotted the behavior of system together after changing initial point and we are seeing the system around 100 seconds, the route of system was changed.

Step 2

The parameters have been set to $a = 3$ $u = 1.3$ and The numerical solution has been approximated from $t = 0$ to $t = 300$. Plot the behavior of system for $(x_1, x_2, x_3) = (1.14531302, 1.14531302, 1.14531302)$.

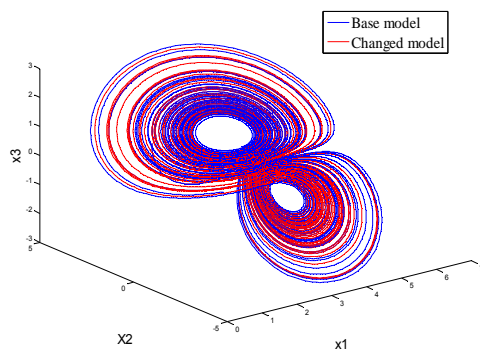


Figure 5. Difference behavior of Rikitake system in 3D plot for second mode

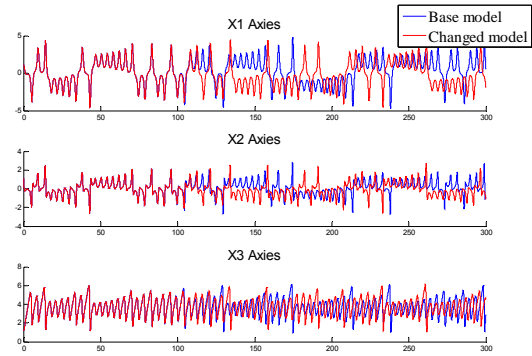


Figure 6. Difference between base and changed model For second mode

After changing the base model (blue color) and changed model (red color) plotted together. We are observing difference between base model and changed model. If we zoom on behavior of system it is clear that the difference occurs before 100 seconds but it is very small and negligible.

Step 3

The parameters have been set to $a = 3$ $u = 1.3$ and The numerical solution has been approximated from $t = 0$ to $t = 300$. Plot the behavior of system for $(x_1, x_2, x_3) = (1.14531303, 1.14531303, 1.14531303)$

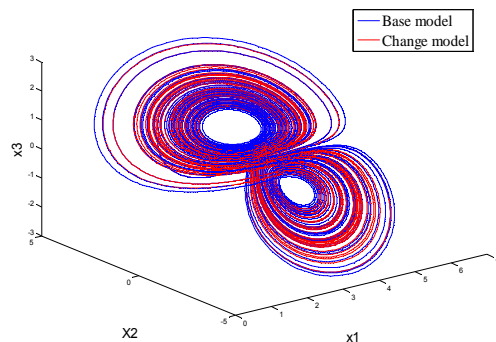


Figure 7. Difference behavior of Rikitake system in 3D plot for third mode

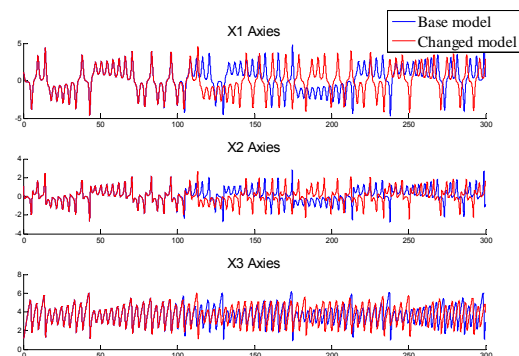


Figure 8. Difference between base and changed model For third mode

After increasing one unit the values of initiation point, similar to the previous two steps the base model and changed model, have found difference. But with this difference that the increasing initiation point the time of creation difference be faster.

3. Conclusion

In this letter we illustrated the Butterfly effect on a chaotic Rikitake system and we were looking to prove the lecture of Edward Lorenz in 1972 with this title "*Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?*" on the Rikitake system. At the first plotted the behavior of system after that changed values of initiation point and observed the Butterfly effect on this system. The first behavior of system was the same but after some seconds differences

appeared and increasing the values of initiation point, caused the time of creation difference be faster. Now we can reclaim “*the flap of a butterfly’s wings in Brazil set off a tornado in Texas*”

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